STATE OF SOUTH CAROLINA

DEPARTMENT OF EDUCATION

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STATE SUPERINTENDENT OF EDUCATION

The South Carolina Department of Education Mathematics Priority and Support Learning Standards

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Contents

Priority Learning Standard Grade Level Count
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Grade Level	# of Math Priority Learning Standards	#of Math Standards
Kindergarten	7	28
First Grade	9	38
Second Grade	12	28
Third Grade	13	38
Fourth Grade	13	36
Fifth Grade	14	41
Sixth Grade	11	69
Seventh Grade	12	80
Eighth Grade	11	104
Foundations of Algebra	17	40
Intermediate Algebra	11	27
Algebra 1	17	41
Algebra 2	9	30
Geometry	17	50
Pre-Calculus	24	62
Calculus	12	27
Probability and Statistics	10	32

*Counts include standards and letter standards.

Rationale for Priority and Support Learning Standards
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Definition
An understanding of how students learn paired with current and relevant numeracy research guided cross grade level teams to analyze the *South Carolina College and Career-Ready Standards for Mathematics* to make decisions on focus mathematical skills. The standards that best represent the focus skills emerged as priority learning standards for each grade level. Ainsworth (2015) states that Standards should play a prioritizing or supporting role for learning in each grade level.

- Priority Standards are "a carefully selected subset of the total list of the grade- specific and course-specific standards within each content area that students must know and be able to do by the end of each school year in order to be prepared for the standards at the next grade level or course. Priority standards represent the assured student competencies that each teacher needs to help every student learn, and demonstrate proficiency in, by the end of the current grade or course" (Ainsworth, 2013, p. xv).
- Support Standards are "those standards that support, connect to, or enhance the Priority Standards. They are taught within the context of the Priority Standards, but do not receive the same degree of instruction and assessment emphasis as do the Priority Standards. The supporting standards often become the instructional scaffolds to help students understand and attain the more rigorous and comprehensive Priority Standards" (Ainsworth, 2013, p. xv).

The priority learning standards are rationalized to articulate the **endurance**, **leverage**, and **readiness** of the standard. The lens of **endurance** represents a standard that communicates a "lasting beyond one grade or course; concepts and skills needed in life" (Ainsworth, 2015). The lens of **leverage** represents a standard that maximizes the value of a concept or skill across academic disciplines. The lens of **readiness** for the next level of learning represents a prerequisite concepts and skills students need to enter a new grade level or course of study" (Ainsworth, 2015).

Purpose
It is important to know that "prioritizing certain standards over others does not mean eliminating those standards that do not make it into the starring roles. All standards must be taught and assessed, and re-taught and reassessed, to gain evidence of student competency of those learning outcomes. Prioritizing the standards has nothing whatsoever to do with "lowering the bar," and everything to do with focus. It is about "less" being more. The difference is in the degree of focus given to certain standards over others" (Ainsworth, 2015).

Organization of Priority and Support Learning Standard
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The priority and support learning standards are organized by grade level for kindergarten through eighth grade. Within the grade levels, priority and support learning standards are grouped by key concepts. Secondary priority and support learning standards are organized by course name.

Each priority learning standard is bold and is noted by a solid bullet. Each support learning standard is noted by a clear bullet. Not all priority learning standards have support learning standards. Some support learning standards have additional supporting standards for instruction. A solid square bullet notes these sub-support learning standards. Not all support learning standards have sub-support learning standards.

See below for an example representing the organization of the priority and support learning standards.

- **Priority Learning Standard**
    - Support Learning Standard
        - Sub-support Learning Standard

Please refer to the *South Carolina College and Career-Ready Standards for Mathematics* to reinforce the priority and support learning standards.

Mathematical Process Standards

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The South Carolina College- and Career-Ready (SCCCR) Mathematical Process Standards demonstrate the ways in which students develop conceptual understanding of mathematical content and apply mathematical skills. As a result, the SCCCR Mathematical Process Standards should be integrated within the SCCCR Content Standards for Mathematics for each grade level and course. Since the process standards drive the pedagogical component of teaching and serve as the means by which students should demonstrate understanding of the content standards, the process standards must be incorporated as an integral part of overall student expectations when assessing content understanding.

Mathematical Process Standards

1. Make sense of problems and persevere in solving them.
- A. Relate a problem to prior knowledge.
- B. Recognize there may be multiple entry points to a problem and more than one path to a solution.
- C. Analyze what is given, what is not given, what is being asked, and what strategies are needed, and make an initial attempt to solve a problem.
- D. Evaluate the success of an approach to solve a problem and refine it if necessary.

2. Reason both contextually and abstractly.
- A. Make sense of quantities and their relationships in mathematical and real-world situations.
- B. Describe a given situation using multiple mathematical representations.
- C. Translate among multiple mathematical representations and compare the meanings each representation conveys about the situation.
- D. Connect the meaning of mathematical operations to the context of a given situation.

3. Use critical thinking skills to justify mathematical reasoning and critique the reasoning of others.
- A. Construct and justify a solution to a problem.
- B. Compare and discuss the validity of various reasoning strategies.
- C. Make conjectures and explore their validity.
- D. Reflect on and provide thoughtful responses to the reasoning of others.

4. Connect mathematical ideas and real-world situations through modeling.
- A. Identify relevant quantities and develop a model to describe their relationships.
- B. Interpret mathematical models in the context of the situation.
- C. Make assumptions and estimates to simplify complicated situations.
- D. Evaluate the reasonableness of a model and refine if necessary.

5. Use a variety of mathematical tools effectively and strategically.
- A. Select and use appropriate tools when solving a mathematical problem.
- B. Use technological tools and other external mathematical resources to explore and deepen understanding of concepts.

6. Communicate mathematically and approach mathematical situations with precision.
- A. Express numerical answers with the degree of precision appropriate for the context of a situation.
- B. Represent numbers in an appropriate form according to the context of the situation.
- C. Use appropriate and precise mathematical language.
- D. Use appropriate units, scales, and labels.

7. Identify and utilize structure and patterns.
- A. Recognize complex mathematical objects as being composed of more than one simple object.
- B. Recognize mathematical repetition in order to make generalizations.
- C. Look for structures to interpret meaning and develop solution strategies.

Kindergarten
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Number Sense

- **K.NS.4 Understand the relationship between number and quantity. Connect counting to cardinality by demonstrating an understanding that:**
  - **a. the last number said tells the number of objects in the set (cardinality);**
  - **b. the number of objects is the same regardless of their arrangement or the order in which they are counted (conservation of number);**
  - **c. each successive number name refers to a quantity that is one more and each previous number name refers to a quantity that is one less.**
    - K.NS.1 Count forward by ones and tens to 100.
    - K.NS.2 Count forward by ones beginning from any number less than 100.
    - K.NS.3 Read numbers from 0 – 20 and represent a number of objects 0 – 20 with a written numeral.
    - K.NS.5 Count a given number of objects from 1 – 20 and connect this sequence in a one-to-one manner.
    - K.NS.9 Identify first through fifth and last positions in a line of objects.

- **K.NS.7 Determine whether the number of up to ten objects in one group is more than, less than, or equal to the number of up to ten objects in another group using matching and counting strategies.**
  - K.NS.8 Compare two written numerals up to 10 using more than, less than, or equal to.
  - K.NS.6 Recognize a quantity of up to ten objects in an organized arrangement (subitizing).

Number Sense and Base Ten

- **K.NSBT.1 Compose and decompose numbers from 11 – 19 separating ten ones from the remaining ones using objects and drawings.**
  - K.NS.6 Recognize a quantity of up to ten objects in an organized arrangement (subitizing).

Algebraic Thinking and Operations

- **K.ATO.2 Solve real-world/story problems using objects and drawings to find sums up to 10 and differences within 10.**
  - K.ATO.1 Model situations that involve addition and subtraction within 10 using objects, fingers, mental images, drawings, acting out situations, verbal explanations, expressions, and equations.
  - K.ATO.3 Compose and decompose numbers up to 10 using objects, drawings, and equations.
  - K.ATO.4 Create a sum of 10 using objects and drawings when given one of two addends 1 – 9.
  - K.ATO.5 Add and subtract fluently within 5.
  - K.NS.6 Recognize a quantity of up to ten objects in an organized arrangement (subitizing).

- K.ATO.6 Describe simple repeating patterns using AB, AAB, ABB, and ABC type patterns.

Geometry

- **K.G.2 Identify and describe a given shape and shapes of objects in everyday situations to include two-dimensional shapes (i.e., triangle, square, rectangle, hexagon, and circle) and three-dimensional shapes (i.e., cone, cube, cylinder, and sphere).**
  - K.G.1 Describe positions of objects by appropriately using terms, including below, above, beside, between, inside, outside, in front of, or behind.
  - K.G.3 Classify shapes as two-dimensional/flat or three-dimensional/solid and explain the reasoning used.
  - K.G.4 Analyze and compare two- and three-dimensional shapes of different sizes and orientations using informal language.
  - K.G.5 Draw two-dimensional shapes (i.e., square, rectangle, triangle, hexagon, and circle) and create models of three-dimensional shapes (i.e., cone, cube, cylinder, and sphere).
  - K.ATO.6 Describe simple repeating patterns using AB, AAB, ABB, and ABC type patterns.

Measurement and Data Analysis

- **K.MDA.2 Compare objects using words such as shorter/longer, shorter/taller, and lighter/heavier.**
  - K.MDA.1 Identify measurable attributes (length, weight) of an object.

- **K.MDA.4 Represent data using objects and picture graphs and draw conclusions from the graphs.**
  - K.MDA.3 Sort and classify data into 2 or 3 categories with data not to exceed 20 items in each category.

First Grade
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Number Sense and Base Ten

- **1.NSBT.1 Extend the number sequence to:**
  - **a. count forward by ones to 120 starting at any number;**
  - **b. count by fives and tens to 100, starting at any number;**
  - **c. read, write and represent numbers to 100 using concrete models, standard form, and equations in expanded form;**
  - **d. read and write in word form numbers zero through nineteen, and multiples often through ninety.**
    - 1.NSBT.3 Compare two two-digit numbers based on the meanings of the tens and ones digits, using the words greater than, equal to, or less than.
    - 1.NSBT.4 Add through 99 using concrete models, drawings, and strategies based on place value to:
      a. add a two-digit number and a one-digit number, understanding that sometimes it is necessary to compose a ten (regroup);
      b. add a two-digit number and a multiple of 10.
    - 1.NSBT.5 Determine the number that is 10 more or 10 less than a given number through 99 and explain the reasoning verbally and with multiple representations, including concrete models.
    - 1.NSBT.6 Subtract a multiple of 10 from a larger multiple of 10, both in the range 10 to 90, using concrete models, drawings, and strategies based on place value.
    - 1.MDA.6 Identify a penny, nickel, dime and quarter and write the coin values using a ¢ symbol.

- **1.NSBT.2 Understand place value through 99 by demonstrating that:**
  - **a. ones can be thought of as a bundle (group) called a "ten";**
  - **b. the tens digit in a two-digit number represents the number of tens and the ones digit represents the number of ones;**
  - **c. two-digit numbers can be decomposed in a variety of ways (e.g., 52 can be decomposed as 5 tens and 2 ones or 4 tens and 12 ones, etc.) and record the decomposition as an equation.**
    - 1.NSBT.3 Compare two two-digit numbers based on the meanings of the tens and ones digits, using the words greater than, equal to, or less than.
    - 1.NSBT.4 Add through 99 using concrete models, drawings, and strategies based on place value to:
      a. add a two-digit number and a one-digit number, understanding that sometimes it is necessary to compose a ten (regroup);
      b. add a two-digit number and a multiple of 10.
    - 1.NSBT.5 Determine the number that is 10 more or 10 less than a given number through 99 and explain the reasoning verbally and with multiple representations, including concrete models.
    - 1.NSBT.6 Subtract a multiple of 10 from a larger multiple of 10, both in the range 10 to 90, using concrete models, drawings, and strategies based on place value.

- 1.MDA.6 Identify a penny, nickel, dime and quarter and write the coin values using a ¢ symbol.

Algebraic Thinking and Operations

- **1.ATO.1 Solve real-world/story problems using addition (as a joining action and as a part-part-whole action) and subtraction (as a separation action, finding parts of the whole, and as a comparison) through 20 with unknowns in all positions.**
  - 1.ATO.2 Solve real-world/story problems that include three whole number addends whose sum is less than or equal to 20.
  - 1.ATO.3 Apply Commutative and Associative Properties of Addition to find the sum (through 20) of two or three addends.
  - 1.ATO.4 Understand subtraction as an unknown addend problem.
  - 1.ATO.5 Recognize how counting relates to addition and subtraction.
  - 1.ATO.6 Demonstrate:
    a. addition and subtraction through 20;
    b. fluency with addition and related subtraction facts through 10.
  - 1.ATO.8 Determine the missing number in addition and subtraction equations within 20.
  - 1.ATO.9 Create, extend and explain using pictures and words for:
    a. repeating patterns (e.g., AB, AAB, ABB, and ABC type patterns);
    b. growing patterns (between 2 and 4 terms/figures).

- **1.ATO.7 Understand the meaning of the equal sign as a relationship between two quantities (sameness) and determine if equations involving addition and subtraction are true.**
  - 1.NSBT.3 Compare two two-digit numbers based on the meanings of the tens and ones digits, using the words greater than, equal to, or less than.
  - 1.NSBT.4 Add through 99 using concrete models, drawings, and strategies based on place value to:
    a. add a two-digit number and a one-digit number, understanding that sometimes it is necessary to compose a ten (regroup);
    b. add a two-digit number and a multiple of 10.
  - 1.NSBT.5 Determine the number that is 10 more or 10 less than a given number through 99 and explain the reasoning verbally and with multiple representations, including concrete models.
  - 1.NSBT.6 Subtract a multiple of 10 from a larger multiple of 10, both in the range 10 to 90, using concrete models, drawings, and strategies based on place value.
  - 1.ATO.4 Understand subtraction as an unknown addend problem.
  - 1.ATO.5 Recognize how counting relates to addition and subtraction.
  - 1.ATO.6 Demonstrate:
    c. addition and subtraction through 20;
    d. fluency with addition and related subtraction facts through 10.
  - 1.ATO.8 Determine the missing number in addition and subtraction equations within 20.
  - 1.MDA.6 Identify a penny, nickel, dime and quarter and write the coin values using a ¢ symbol.

Geometry

- **1.G.1 Distinguish between a two-dimensional shape's defining (e.g., number of sides) and non-defining attributes (e.g., color).**
    - 1.G.2 Combine two-dimensional shapes (i.e., square, rectangle, triangle, hexagon, rhombus, and trapezoid) or three-dimensional shapes (i.e., cube, rectangular prism, cone, and cylinder) in more than one way to form a composite shape.
    - 1.G.4 Identify and name two-dimensional shapes (i.e., square, rectangle, triangle, hexagon, rhombus, trapezoid, and circle).

- **1.G.3 Partition two-dimensional shapes (i.e., square, rectangle, circle) into two or four equal parts.**
    - 1.G.2 Combine two-dimensional shapes (i.e., square, rectangle, triangle, hexagon, rhombus, and trapezoid) or three-dimensional shapes (i.e., cube, rectangular prism, cone, and cylinder) in more than one way to form a composite shape.
    - 1.G.4 Identify and name two-dimensional shapes (i.e., square, rectangle, triangle, hexagon, rhombus, trapezoid, and circle).

Measurement and Data Analysis

- **1.MDA.2 Use nonstandard physical models to show the length of an object as the number of same size units of length with no gaps or overlaps.**
    - 1.MDA.1 Order three objects by length using indirect comparison.
    - 1.ATO.5 Recognize how counting relates to addition and subtraction.

- **1.MDA.3 Use analog and digital clocks to tell and record time to the hour and half hour.**
    - 1.G.3 Partition two-dimensional shapes (i.e., square, rectangle, circle) into two or four equal parts.

- **1.MDA.5 Draw conclusions from given object graphs, picture graphs, t-charts, tallies, and bar graphs.**
    - 1.MDA.4 Collect, organize, and represent data with up to 3 categories using object graphs, picture graphs, t-charts and tallies.
    - 1.MDA.6 Identify a penny, nickel, dime and quarter and write the coin values using a ¢ symbol.
    - 1.ATO.9 Create, extend and explain using pictures and words for:
        a. repeating patterns (e.g., AB, AAB, ABB, and ABC type patterns);
        b. growing patterns (between 2 and 4 terms/figures).

Second Grade
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Number Sense and Base Ten

- **2.NSBT.3 Read, write and represent numbers through 999 using concrete models, standard form, and equations in expanded form.**
    - 2.NSBT.1 Understand place value through 999 by demonstrating that:
        a. 100 can be thought of as a bundle (group) of 10 tens called a "hundred";
        b. the hundreds digit in a three-digit number represents the number of hundreds, the tens digit represents the number of tens, and the ones digit represents the number of ones;
        c. three-digit numbers can be decomposed in multiple ways (e.g., 524 can be decomposed as 5 hundreds, 2 tens and 4 ones or 4 hundreds, 12 tens, and 4 ones, etc.).
    - 2.NSBT.2 Count by tens and hundreds to 1,000 starting with any number.
    - 2.NSBT.5 Add and subtract fluently through 99 using knowledge of place value and properties of operations.
    - 2.NSBT.6 Add up to four two-digit numbers using strategies based on knowledge of place value and properties of operations.
    - 2.ATO.3 Determine whether a number through 20 is odd or even using pairings of objects, counting by twos, or finding two equal addends to represent the number (e.g., 3 + 3 = 6).
    - 2.MDA.7 Solve real-world/story problems involving dollar bills using the $ symbol or involving quarters, dimes, nickels, and pennies using the ¢ symbol.

- **2.NSBT.4 Compare two numbers with up to three digits using words and symbols (i.e., >, =, or <).**
    - 2.NSBT.8 Determine the number that is 10 or 100 more or less than a given number through 1,000 and explain the reasoning verbally and in writing.
    - 2.MDA.7 Solve real-world/story problems involving dollar bills using the $ symbol or involving quarters, dimes, nickels, and pennies using the ¢ symbol.

- 2.NSBT.7 Add and subtract through 999 using concrete models, drawings, and symbols which convey strategies connected to place value understanding.
    - 2.NSBT.5 Add and subtract fluently through 99 using knowledge of place value and properties of operations.
    - 2.NSBT.6 Add up to four two-digit numbers using strategies based on knowledge of place value and properties of operations.
    - 2.MDA.5 Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, …, and represent whole-number sums and differences through 99 on a number line diagram.
    - 2.MDA.7 Solve real-world/story problems involving dollar bills using the $ symbol or involving quarters, dimes, nickels, and pennies using the ¢ symbol.

Algebraic Thinking and Operations

- **2.ATO.1 Solve one- and two-step real-world/story problems using addition (as a joining action and as a part-part-whole action) and subtraction (as a separation action, finding parts of the whole, and as a comparison) through 99 with unknowns in all positions.**
    - 2.NSBT.5 Add and subtract fluently through 99 using knowledge of place value and properties of operations.
    - 2.ATO.2 Demonstrate fluency with addition and related subtraction facts through 20.

- **2.ATO.4 Use repeated addition to find the total number of objects arranged in a rectangular array with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.**

Geometry

- **2.G.1 Identify triangles, quadrilaterals, hexagons, and cubes. Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces.**

- **2.G.2 Partition a rectangle into rows and columns of same-size squares to form an array and count to find the total number of parts.**

- **2.G.3 Partition squares, rectangles and circles into two or four equal parts, and describe the parts using the words halves, fourths, a half of, and a fourth of. Understand that when partitioning a square, rectangle or circle into two or four equal parts, the parts become smaller as the number of parts increases.**

Measurement and Data Analysis

- **2.MDA.1 Select and use appropriate tools (e.g., rulers, yardsticks, meter sticks, measuring tapes) to measure the length of an object.**
    - 2.MDA.2 Measure the same object or distance using a standard unit of one length and then a standard unit of a different length and explain verbally and in writing how and why the measurements differ.
    - 2.MDA.3 Estimate and measure length/distance in customary units (i.e., inch, foot, yard) and metric units (i.e., centimeter, meter).
    - 2.MDA.4 Measure to determine how much longer one object is than another, using standard length units.
    - 2.MDA.8 Generate data by measuring objects in whole unit lengths and organize the data in a line plot using a horizontal scale marked in whole number units.

- **2.MDA.5 Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, …, and represent whole-number sums and differences through 99 on a number line diagram.**
    - 2.MDA.4 Measure to determine how much longer one object is than another, using standard length units.

    - 2.MDA.8 Generate data by measuring objects in whole unit lengths and organize the data in a line plot using a horizontal scale marked in whole number units.

- **2.MDA.6 Use analog and digital clocks to tell and record time to the nearest five-minute interval using a.m. and p.m.**

- **2.MDA.10 Draw conclusions from t-charts, object graphs, picture graphs, and bar graphs.**
    - 2.ATO.2 Demonstrate fluency with addition and related subtraction facts through 20.
    - 2.MDA.7 Solve real-world/story problems involving dollar bills using the $ symbol or involving quarters, dimes, nickels, and pennies using the ¢ symbol.
    - 2.MDA.9 Collect, organize, and represent data with up to four categories using picture graphs and bar graphs with a single-unit scale

Third Grade
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Number Sense and Base Ten

- **3.NSBT.1 Use place value understanding to round whole numbers to the nearest 10 or 100.**
    - 3.NSBT.4 Read and write numbers through 999,999 in standard form and equations in expanded form.

- **3.NSBT.5 Compare and order numbers through 999,999 and represent the comparison using the symbols >, =, or <.**
    - 3.NSBT.4 Read and write numbers through 999,999 in standard form and equations in expanded form.

Number Sense - Fractions

- **3.NSF.1 Develop an understanding of fractions (i.e., denominators 2, 3, 4, 6, 8, 10) as numbers.**
    - **a. A fraction $\frac{1}{b}$ (called a unit fraction) is the quantity formed by one part when a whole is partitioned into b equal parts;**
    - **b. A fraction $\frac{a}{b}$ is the quantity formed by a parts of size $\frac{1}{b}$;**
    - **c. A fraction is a number that can be represented on a number line based on counts of a unit fraction;**
    - **d. A fraction can be represented using set, area, and linear models.**
        - 3.G.2 Partition two-dimensional shapes into 2, 3, 4, 6, or 8 parts with equal areas and express the area of each part using the same unit fraction. Recognize that equal parts of identical wholes need not have the same shape.
        - 3.MDA.2 Estimate and measure liquid volumes (capacity) in customary units (i.e., c., pt., qt., gal.) and metric units (i.e., mL, L) to the nearest whole unit.
        - 3.MDA.4 Generate data by measuring length to the nearest inch, half-inch and quarter-inch and organize the data in a line plot using a horizontal scale marked off in appropriate units.

- **3.NSF.2 Explain fraction equivalence (i.e., denominators 2, 3, 4, 6, 8, 10) by demonstrating an understanding that:**
    - **a. two fractions are equal if they are the same size, based on the same whole, or at the same point on a number line;**
    - **b. fraction equivalence can be represented using set, area, and linear models;**
    - **c. whole numbers can be written as fractions (e.g., $4=\frac{4}{1}$ and $1=\frac{4}{4}$);**
    - **d. fractions with the same numerator or same denominator can be compared by reasoning about their size based on the same whole.**
        - 3.MDA.4 Generate data by measuring length to the nearest inch, half-inch and quarter-inch and organize the data in a line plot using a horizontal scale marked off in appropriate units.

- **3.NSF.3 Develop an understanding of mixed numbers (ie., denominators 2, 3, 4, 6, 8, 10) as iterations of unit fractions on a number line.**
    - 3.MDA.2 Estimate and measure liquid volumes (capacity) in customary units (i.e., c., pt., qt., gal.) and metric units (i.e., mL, L) to the nearest whole unit.
    - 3.MDA.4 Generate data by measuring length to the nearest inch, half-inch and quarter-inch and organize the data in a line plot using a horizontal scale marked off in appropriate units.

Algebraic Thinking and Operations

- **3.ATO.3 Solve real-world problems involving equal groups, area/array, and number line models using basic multiplication and related division facts. Represent the problem situation using an equation with a symbol for the unknown.**
    - 3.ATO.1 Use concrete objects, drawings and symbols to represent multiplication facts of two single-digit whole numbers and explain the relationship between the factors (i.e., 0 – 10) and the product.
    - 3.ATO.2 Use concrete objects, drawings and symbols to represent division without remainders and explain the relationship among the whole number quotient (i.e., 0 – 10), divisor (i.e., 0 – 10), and dividend.
    - 3.ATO.4 Determine the unknown whole number in a multiplication or division equation relating three whole numbers when the unknown is a missing factor, product, dividend, divisor, or quotient.
    - 3.ATO.5 Apply properties of operations (i.e., Commutative Property of Multiplication, Associative Property of Multiplication, Distributive Property) as strategies to multiply and divide and explain the reasoning.
    - 3.ATO.6 Understand division as a missing factor problem.
    - 3.ATO.7 Demonstrate fluency with basic multiplication and related division facts of products and dividends through 100.
    - 3.ATO.9 Identify a rule for an arithmetic pattern (e.g., patterns in the addition table or multiplication table).

- **3.ATO.8 Solve two-step real-world problems using addition, subtraction, multiplication and division of whole numbers and having whole number answers. Represent these problems using equations with a letter for the unknown quantity.**
    - 3.NSBT.2 Add and subtract whole numbers fluently to 1,000 using knowledge of place value and properties of operations.
    - 3.NSBT.3 Multiply one-digit whole numbers by multiples of 10 in the range 10 – 90, using knowledge of place value and properties of operations.
    - 3.ATO.1 Use concrete objects, drawings and symbols to represent multiplication facts of two single-digit whole numbers and explain the relationship between the factors (i.e., 0 – 10) and the product.
    - 3.ATO.2 Use concrete objects, drawings and symbols to represent division without remainders and explain the relationship among the whole number quotient (i.e., 0 – 10), divisor (i.e., 0 – 10), and dividend.
    - 3.ATO.4 Determine the unknown whole number in a multiplication or division equation relating three whole numbers when the unknown is a missing factor, product, dividend, divisor, or quotient.

- 3.ATO.5 Apply properties of operations (i.e., Commutative Property of Multiplication, Associative Property of Multiplication, Distributive Property) as strategies to multiply and divide and explain the reasoning.
- 3.ATO.6 Understand division as a missing factor problem.
- 3.ATO.7 Demonstrate fluency with basic multiplication and related division facts of products and dividends through 100.
- 3.ATO.9 Identify a rule for an arithmetic pattern (e.g., patterns in the addition table or multiplication table).

Geometry

- **3.G.1 Understand that shapes in different categories (e.g., rhombus, rectangle, square, and other 4-sided shapes) may share attributes (e.g., 4- sided figures) and the shared attributes can define a larger category (e.g., quadrilateral). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.**
  - 3.G.4 Identify a three-dimensional shape (i.e., right rectangular prism, right triangular prism, pyramid) based on a given two-dimensional net and explain the relationship between the shape and the net.

- **3.G.2 Partition two-dimensional shapes into 2, 3, 4, 6, or 8 parts with equal areas and express the area of each part using the same unit fraction. Recognize that equal parts of identical wholes need not have the same shape.**

- **3.G.3 Use a right angle as a benchmark to identify and sketch acute and obtuse angles.**

Measurement and Data Analysis

- **3.MDA.1 Use analog and digital clocks to determine and record time to the nearest minute, using a.m. and p.m.; measure time intervals in minutes; and solve problems involving addition and subtraction of time intervals within 60 minutes.**

- **3.MDA.3 Collect, organize, classify, and interpret data with multiple categories and draw a scaled picture graph and a scaled bar graph to represent the data.**

- **3.MDA.6 Solve real-world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.**
  - 3.MDA.5 Understand the concept of area measurement.
    a. Recognize area as an attribute of plane figures;
    b. Measure area by building arrays and counting standard unit squares;
    c. Determine the area of a rectilinear polygon and relate to multiplication and addition.

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Number Sense and Base Ten

- **4.NSBT.1 Understand that, in a multi-digit whole number, a digit represents ten times what the same digit represents in the place to its right.**
    - 4.NSBT.2 Recognize math periods and number patterns within each period to read and write in standard form large numbers through 999,999,999.

- **4.NSBT.5 Multiply up to a four-digit number by a one-digit number and multiply a two-digit number by a two-digit number using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using rectangular arrays, area models and/or equations.**
    - 4.NSBT.4 Fluently add and subtract multi-digit whole numbers using strategies to include a standard algorithm.
    - 4.NSBT.3 Use rounding as one form of estimation and round whole numbers to any given place value.

- **4.NSBT.6 Divide up to a four-digit dividend by a one-digit divisor using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division.**
    - 4.NSBT.4 Fluently add and subtract multi-digit whole numbers using strategies to include a standard algorithm.
    - 4.NSBT.3 Use rounding as one form of estimation and round whole numbers to any given place value.

Number Sense and Operations - Fractions

- **4.NSF.1 Explain why a fraction (i.e., denominators 2, 3, 4, 5, 6, 8, 10, 12, 25, 100),** $\frac{a}{b}$**, is equivalent to a fraction,** $\frac{n \times a}{n \times b}$**, by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.**
    - 4.NSF.2 Compare two given fractions (i.e., denominators 2, 3, 4, 5, 6, 8, 10, 12, 25, 100) by creating common denominators or numerators, or by comparing to a benchmark fraction such as $\frac{1}{2}$ and represent the comparison using the symbols >, =, or <.
    - 4.NSF.5 Express a fraction with a denominator of 10 as an equivalent fraction with a denominator of 100 and use this technique to add two fractions with respective denominators of 10 and 100.
    - 4.NSF.6 Write a fraction with a denominator of 10 or 100 using decimal notation, and read and write a decimal number as a fraction.
    - 4.NSF.7 Compare and order decimal numbers to hundredths, and justify using concrete and visual models.
    - 4.MDA.4 Create a line plot to display a data set (i.e., generated by measuring length to the nearest quarter-inch and eighth-inch) and interpret the line plot.

- **4.NSF.3 Develop an understanding of addition and subtraction of fractions (i.e., denominators 2, 3, 4, 5, 6, 8, 10, 12, 25, 100) based on unit fractions.**
  - **a. Compose and decompose a fraction in more than one way, recording each composition and decomposition as an addition or subtraction equation;**
  - **b. Add and subtract mixed numbers with like denominators;**
  - **c. Solve real-world problems involving addition and subtraction of fractions referring to the same whole and having like denominators.**
    - 4.NSF.5 Express a fraction with a denominator of 10 as an equivalent fraction with a denominator of 100 and use this technique to add two fractions with respective denominators of 10 and 100.
    - 4.NSBT.4 Fluently add and subtract multi-digit whole numbers using strategies to include a standard algorithm.

- **4.NSF.4 Apply and extend an understanding of multiplication by multiplying a whole number and a fraction (i.e., denominators 2, 3, 4, 5, 6, 8, 10, 12, 25, 100).**
  - **a. Understand a fraction $\frac{a}{b}$ as a multiple of $\frac{1}{b}$;**
  - **b. Understand a multiple of $\frac{a}{b}$ as a multiple of $\frac{1}{b}$, and use this understanding to multiply a fraction by a whole number;**
  - **c. Solve real-world problems involving multiplication of a fraction by a whole number (i.e., use visual fraction models and equations to represent the problem).**
    - 4.NSBT.4 Fluently add and subtract multi-digit whole numbers using strategies to include a standard algorithm.

Algebraic Thinking and Operations

- **4.ATO.3 Solve multi-step, real-world problems using the four operations. Represent the problem using an equation with a variable as the unknown quantity.**
  - 4.ATO.1 Interpret a multiplication equation as a comparison (e.g. interpret 35= 5x7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5.) Represent verbal statements of multiplicative comparisons as multiplication equations.
  - 4.ATO.2 Solve real-world problems using multiplication (product unknown) and division (group size unknown, number of groups unknown).
  - 4.ATO.4 Recognize that a whole number is a multiple of each of its factors. Find all factors for a whole number in the range 1 – 100 and determine whether the whole number is prime or composite.
  - 4.ATO.5 Generate a number or shape pattern that follows a given rule and determine a term that appears later in the sequence.
  - 4.MDA.8 Determine the value of a collection of coins and bills greater than $1.00.

Geometry

- **4.G.2 Classify quadrilaterals based on the presence or absence of parallel or perpendicular lines.**
    - 4.G.1 Draw points, lines, line segments, rays, angles (i.e., right, acute, obtuse), and parallel and perpendicular lines. Identify these in two- dimensional figures.

- **4.G.3 Recognize right triangles as a category, and identify right triangles.**
    - 4.G.1 Draw points, lines, line segments, rays, angles (i.e., right, acute, obtuse), and parallel and perpendicular lines. Identify these in two- dimensional figures.
    - 4.MDA.5 Understand the relationship of an angle measurement to a circle.
    - 4.MDA.6 Measure and draw angles in whole number degrees using a protractor.

4.G.4 Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.

Measurement and Data Analysis

- **4.MDA.2 Solve real-world problems involving distance/length, intervals of time within 12 hours, liquid volume, mass, and money using the four operations.**
    - 4.ATO.2 Solve real-world problems using multiplication (product unknown) and division (group size unknown, number of groups unknown).
    - 4.MDA.1 Convert measurements within a single system of measurement, customary (i.e., in., ft., yd., oz., lb., sec., min., hr.) or metric (i.e., cm, m, km, g, kg, mL, L) from a larger to a smaller unit.
    - 4.MDA.4 Create a line plot to display a data set (i.e., generated by measuring length to the nearest quarter-inch and eighth-inch) and interpret the line plot.
    - 4.MDA.8 Determine the value of a collection of coins and bills greater than $1.00.

- **4.MDA.3 Apply the area and perimeter formulas for rectangles.**
    - 4.ATO.2 Solve real-world problems using multiplication (product unknown) and division (group size unknown, number of groups unknown).

- **4.MDA.7 Solve addition and subtraction problems to find unknown angles in real-world and mathematical problems.**
    - 4.MDA.6 Measure and draw angles in whole number degrees using a protractor.

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Number Sense and Base Ten

- **5. NSBT.1 Understand that, in a multi-digit whole number, a digit in one place represents 10 times what the same digit represents in the place to its right, and represents $\frac{1}{10}$ times what the same digit represents in the place to its left.**
    - 5.NSBT.2 Use whole number exponents to explain:
        a. patterns in the number of zeroes of the product when multiplying a number by powers of 10;
        b. patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10.

- **5. NSBT.3 Read and write decimals in standard and expanded form. Compare two decimal numbers to the thousandths using the symbols >, =, or <.**

- **5. NSBT.7 Add, subtract, multiply, and divide decimal numbers to hundredths using concrete area models and drawings.**
    - 5. NSBT.4 Round decimals to any given place value within thousandths.
    - 5. NSBT.5 Fluently multiply multi-digit whole numbers using strategies to include a standard algorithm.
    - 5. NSBT.6 Divide up to a four-digit dividend by a two-digit divisor, using strategies based on place value, the properties of operations, and the relationship between multiplication and division.

Number Sense and Operations - Fractions

- **5. NSF.2 Solve real-world problems involving addition and subtraction of fractions with unlike denominators.**
    - 5. NSF.1 Add and subtract fractions with unlike denominators (including mixed numbers) using a variety of models, including an area model and number line.

- **5. NSF.6 Solve real-world problems involving multiplication of a fraction by a fraction, improper fraction and a mixed number.**
    - 5. NSF.4 Extend the concept of multiplication to multiply a fraction or whole number by a fraction.
        a. Recognize the relationship between multiplying fractions and finding the areas of rectangles with fractional side lengths;
        b. Interpret multiplication of a fraction by a whole number and a whole number by a fraction and compute the product;
        c. Interpret multiplication in which both factors are fractions less than one and compute the product.
    - 5. NSF.5 Justify the reasonableness of a product when multiplying with fractions.
        a. Estimate the size of the product based on the size of the two factors;

b. Explain why multiplying a given number by a number greater than 1 (e.g., improper fractions, mixed numbers, whole numbers) results in a product larger than the given number;
c. Explain why multiplying a given number by a fraction less than 1 results in a product smaller than the given number;
d. Explain why multiplying the numerator and denominator by the same number has the same effect as multiplying the fraction by 1.

- **5.NSF.8 Solve real-world problems involving division of unit fractions and whole numbers, using visual fraction models and equations.**
  - 5.NSF.3 Understand the relationship between fractions and division of whole numbers by interpreting a fraction as the numerator divided by the denominator (i.e., $\frac{a}{b} = a \div b$).
  - 5.NSF.7 Extend the concept of division to divide unit fractions and whole numbers by using visual fraction models and equations.
    a. Interpret division of a unit fraction by a non-zero whole number and compute the quotient;
    b. Interpret division of a whole number by a unit fraction and compute the quotient.

Algebraic Thinking and Operations

- **5.ATO.1 Evaluate numerical expressions involving grouping symbols (i.e., parentheses, brackets, braces).**

- **5.ATO.2 Translate verbal phrases into numerical expressions and interpret numerical expressions as verbal phrases.**

- **5.ATO.3 Investigate the relationship between two numerical patterns.**
  a. Generate two numerical patterns given two rules and organize in tables;
  b. Translate the two numerical patterns into two sets of ordered pairs;
  c. Graph the two sets of ordered pairs on the same coordinate plane;
  d. Identify the relationship between the two numerical patterns.

Geometry

- **5.G.2 Plot and interpret points in the first quadrant of the coordinate plane to represent real-world and mathematical situations.**
  - 5.G.1 Define a coordinate system.
    a. The x- and y- axes are perpendicular number lines that intersect at 0 (the origin);
    b. Any point on the coordinate plane can be represented by its coordinates;
    c. The first number in an ordered pair is the x-coordinate and represents the horizontal distance from the origin;
    d. The second number in an ordered pair is the y-coordinate and represents the vertical distance from the origin.

- **5.G.4 Classify two-dimensional figures in a hierarchy based on their attributes.**
    - 5.G.3 Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category.

Measurement and Data Analysis

- **5.MDA.1 Convert measurements within a single system of measurement: customary (i.e., in., ft., yd., oz., lb., sec., min., hr.) or metric (i.e., mm, cm, m, km, g, kg, mL, L) from a larger to a smaller unit and a smaller to a larger unit.**
    - 5.NSBT.5 Fluently multiply multi-digit whole numbers using strategies to include a standard algorithm.
    - 5.NSBT.6 Divide up to a four-digit dividend by a two-digit divisor, using strategies based on place value, the properties of operations, and the relationship between multiplication and division.

- **5.MDA.2 Create a line plot consisting of unit fractions and use operations on fractions to solve problems related to the line plot.**
    - 5.NSF.1 Add and subtract fractions with unlike denominators (including mixed numbers) using a variety of models, including an area model and number line.

- **5.MDA.4 Differentiate among perimeter, area and volume and identify which application is appropriate for a given situation.**
    - 5.MDA.3 Understand the concept of volume measurement.
        a. Recognize volume as an attribute of right rectangular prisms;
        b. Relate volume measurement to the operations of multiplication and addition by packing right rectangular prisms and then counting the layers of standard unit cubes;
        c. Determine the volume of right rectangular prisms using the formula derived from packing right rectangular prisms and counting the layers of standard unit cubes.

6th Grade
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The Number System

- **6.NS.9 Investigate and translate among multiple representations of rational numbers (fractions, decimal numbers, percentages). Fractions should be limited to those with denominators of 2, 3, 4, 5, 8, 10, and 100.**
    - 6.NS.6 Extend the understanding of the number line to include all rational numbers and apply this concept to the coordinate plane.
        - 6.NS.6d. Plot rational numbers on number lines and ordered pairs on coordinate planes.
    - 6.NS.7 Understand and apply the concepts of comparing, ordering, and finding absolute value to rational numbers.
        - 6.NS.7a. Interpret statements using equal to (=) and not equal to (≠).
        - 6.NS.7b. Interpret statements using less than (<), greater than (>), and equal to (=) as relative locations on the number line.
        - 6.NS.7c. Use concepts of equality and inequality to write and to explain real- world and mathematical situations.
        - 6.NS.7d. Understand that absolute value represents a number's distance from zero on the number line and use the absolute value of a rational number to represent real-world situations.
        - 6.NS.7e. Recognize the difference between comparing absolute values and ordering rational numbers. For negative rational numbers, understand that as the absolute value increases, the value of the negative number decreases.

- **6.NS.5 Understand that the positive and negative representations of a number are opposites in direction and value. Use integers to represent quantities in real- world situations and explain the meaning of zero in each situation.**
    - 6.NS.6 Extend the understanding of the number line to include all rational numbers and apply this concept to the coordinate plane.
        - 6.NS.6a. Understand the concept of opposite numbers, including zero, and their relative locations on the number line.
        - 6.NS.6b. Understand that the signs of the coordinates in ordered pairs indicate their location on an axis or in a quadrant on the coordinate plane.
        - 6.NS.6d. Plot rational numbers on number lines and ordered pairs on coordinate planes.
    - 6.NS.7 Understand and apply the concepts of comparing, ordering, and finding absolute value to rational numbers.
        - 6.NS.7a. Interpret statements using equal to (=) and not equal to (≠).
        - 6.NS.7b. Interpret statements using less than (<), greater than (>), and equal to (=) as relative locations on the number line.
        - 6.NS.7c. Use concepts of equality and inequality to write and to explain real-world and mathematical situations.

        - 6.NS.7d. Understand that absolute value represents a number's distance from zero on the number line and use the absolute value of a rational number to represent real-world situations.
        - 6. NS.7e Recognize the difference between comparing absolute values and ordering rational numbers. For negative rational numbers, understand that as the absolute value increases, the value of the negative number decreases.

Expressions, Equations, and Equalities

- **6.EEI.4 Apply mathematical properties (e.g., commutative, associative, distributive) to justify that two expressions are equivalent.**
    - 6.EEI.3 Apply mathematical properties (e.g., commutative, associative, distributive) to generate equivalent expressions.
    - 6.NS.4 Find common factors and multiples using two whole numbers.
        - 6.NS.4a. Compute the greatest common factor (GCF) of two numbers both less than or equal to 100.
        - 6.NS.4b Compute the least common multiple (LCM) of two numbers both less than or equal to 12.
        - 6.NS.4c. Express sums of two whole numbers, each less than or equal to 100, using the distributive property to factor out a common factor of the original addends.
    - 6.NS.2 Fluently divide multi-digit whole numbers using a standard algorithmic approach.
    - 6.NS.1 Compute and represent quotients of positive fractions using a variety of procedures (e.g., visual models, equations, and real-world situations).
    - 6.NS.3 Fluently add, subtract, multiply and divide multi-digit decimal numbers using a standard algorithmic approach.

- **6.EEI.9 Investigate multiple representations of relationships in real-world and mathematical situations.**
    - 6.EEI.9a. Write an equation that models a relationship between independent and dependent variables.
    - 6.EEI.9b. Analyze the relationship between independent and dependent variables using graphs and tables.
    - 6.EEI.9c. Translate among graphs, tables, and equations.
    - 6.EEI.6 Write expressions using variables to represent quantities in real- world and mathematical situations. Understand the meaning of the variable in the context of the situation.

- **6.EEI.8 Extend knowledge of inequalities used to compare numerical expressions to include algebraic expressions in real-world and mathematical situations.**
    - 6.EEI.8a. Write an inequality of the form $x>c$ or $x<c$ and graph the solution set on a number line.
    - 6.EEI.8b. Recognize that inequalities have infinitely many solutions.
    - 6.EEI.5 Understand that if any solutions exist, the solution set for an equation or inequality consists of values that make the equation or inequality true.

- 6.NS.7 Understand and apply the concepts of comparing, ordering, and finding absolute value to rational numbers.
    - 6.NS.7a. Interpret statements using equal to (=) and not equal to (≠).
    - 6.NS.7b. Interpret statements using less than (<), greater than (>), and
    - equal to (=) as relative locations on the number line.
    - 6.NS.7c. Use concepts of equality and inequality to write and to explain real-world and mathematical situations.

- **6.EEI.7 Write and solve one-step linear equations in one variable involving non negative rational numbers for real-world and mathematical situations.**
    - 6.EEI.1 Write and evaluate numerical expressions involving whole-number exponents and positive rational number bases using the Order of Operations.
    - 6.EEI.2 Extend the concepts of numerical expressions to algebraic expressions involving positive rational numbers.
        - 6.EEI.2a. Translate between algebraic expressions and verbal phrases that include variables.
        - 6.EEI.2b. Investigate and identify parts of algebraic expressions using mathematical terminology, including term, coefficient, constant, and factor.
        - 6.EEI.2c. Evaluate real-world and algebraic expressions for specific values using the Order of Operations. Grouping symbols should be limited to parentheses, braces, and brackets. Exponents should be limited to whole- numbers.
    - 6.NS.2 Fluently divide multi-digit whole numbers using a standard algorithmic approach.
    - 6.NS.1 Compute and represent quotients of positive fractions using a variety of procedures (e.g., visual models, equations, and real-world situations).
    - 6.NS.3 Fluently add, subtract, multiply and divide multi-digit decimal numbers using a standard algorithmic approach.

Ratios and Proportional Relationships

- **6.RP.3 Apply the concepts of ratios and rates to solve real-world and mathematical problems.**
    - 6.RP.3a. Create a table consisting of equivalent ratios and plot the results on the coordinate plane.
    - 6.RP.3b. Use multiple representations, including tape diagrams, tables, double number lines, and equations, to find missing values of equivalent ratios.
    - 6.RP.3c. Use two tables to compare related ratios.
    - 6.RP.3d. Apply concepts of unit rate to solve problems, including unit pricing and constant speed.
    - 6.RP.3e. Understand that a percentage is a rate per 100 and use this to solve problems involving wholes, parts, and percentages.
    - 6.RP.3f. Solve one-step problems involving ratios and unit rates (e.g., dimensional analysis).
    - 6.RP.1 Interpret the concept of a ratio as the relationship between two quantities, including part to part and part to whole.

- 6.RP.2 Investigate relationships between ratios and rates.
  - 6.RP.2a. Translate between multiple representations of ratios (i.e., $\frac{a}{b}$, $a{:}b$, a to b, visual models).
  - 6.RP.2b. Recognize that a rate is a type of ratio involving two different units.
  - 6.RP.2c. Convert from rates to unit rates.

Geometry and Measurement

- **6.GM.3 Apply the concepts of polygons and the coordinate plane to real-world and mathematical situations.**
  - 6.GM.3a. Given coordinates of the vertices, draw a polygon in the coordinate plane.
  - 6.GM.3b. Find the length of an edge if the vertices have the same x- coordinates or same y-coordinates.
  - 6.NS.6 Extend the understanding of the number line to include all rational numbers and apply this concept to the coordinate plane.
    - 6.NS.6b. Understand that the signs of the coordinates in ordered pairs indicate their location on an axis or in a quadrant on the coordinate plane.
    - 6.NS.6c. Recognize when ordered pairs are reflections of each other on the coordinate plane across one axis, both axes, or the origin.
    - 6.NS.6d. Plot rational numbers on number lines and ordered pairs on coordinate planes.
  - 6.NS.8 Extend knowledge of the coordinate plane to solve real-world and mathematical problems involving rational numbers.
    - 6.NS.8a. Plot points in all four quadrants to represent the problem.
    - 6.NS.8b. Find the distance between two points when ordered pairs have the same x-coordinates or same y-coordinates.
    - 6.NS.8c. Relate finding the distance between two points in a coordinate plane to absolute value using a number line.
  - 6.GM.1 Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.
  - 6.RP.3f. Solve one-step problems involving ratios and unit rates (e.g., dimensional analysis).
  - 6.DS.5b. Describe the qualitative aspects of the data (e.g., how it was measured, units of measurement).

- **6.GM.4 Unfold three-dimensional figures into two-dimensional rectangles and triangles (nets) to find the surface area and to solve real-world and mathematical problems.**
  - 6.GM.1 Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

- 6.RP.3f. Solve one-step problems involving ratios and unit rates (e.g., dimensional analysis).
- 6.DS.5b. Describe the qualitative aspects of the data (e.g., how it was measured, units of measurement).
- 6.NS.2 Fluently divide multi-digit whole numbers using a standard algorithmic approach.
- 6.NS.1 Compute and represent quotients of positive fractions using a variety of procedures (e.g., visual models, equations, and real-world situations).
- 6.NS.3 Fluently add, subtract, multiply and divide multi-digit decimal numbers using a standard algorithmic approach.

- **6.GM.2 Use visual models (e.g., model by packing) to discover that the formulas for the volume of a right rectangular prism ($V = lwh$, $V = Bh$) are the same for whole or fractional edge lengths. Apply these formulas to solve real-world and mathematical problems.**
  - 6.RP.3f. Solve one-step problems involving ratios and unit rates (e.g., dimensional analysis).
  - 6.DS.5b. Describe the qualitative aspects of the data (e.g., how it was measured, units of measurement).
  - 6.NS.2 Fluently divide multi-digit whole numbers using a standard algorithmic approach.
  - 6.NS.1 Compute and represent quotients of positive fractions using a variety of procedures (e.g., visual models, equations, and real-world situations).
  - 6.NS.3 Fluently add, subtract, multiply and divide multi-digit decimal numbers using a standard algorithmic approach.

Data Analysis and Statistics

- **6.DS.2 Use center (mean, median, mode), spread (range, interquartile range, mean absolute value), and shape (symmetrical, skewed left, skewed right) to describe the distribution of a set of data collected to answer a statistical question.**
  - 6.DS.1 Differentiate between statistical and non-statistical questions.
  - 6.DS.5 Describe numerical data sets in relation to their real-world context.
    - 6.DS.5a. State the sample size.
    - 6.DS.5b. Describe the qualitative aspects of the data (e.g., how it was measured, units of measurement).
    - 6.DS.5c. Give measures of center (median, mean).
    - 6.DS.5d. Find measures of variability (interquartile range, mean absolute deviation) using a number line.
    - 6.DS.5e. Describe the overall pattern (shape) of the distribution.
    - 6.DS.5f. Justify the choices for measure of center and measure of variability based on the shape of the distribution.
    - 6.DS.5g. Describe the impact that inserting or deleting a data point has on the measures of center (median, mean) for a data set.
  - 6.DS.3 Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.

- 6.DS.4 Select and create an appropriate display for numerical data, including dot plots, histograms, and box plots

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The Number System

- **7.NS.5 Extend prior knowledge to translate among multiple representations of rational numbers (fractions, decimal numbers, percentages). Exclude the conversion of repeating decimal numbers to fractions.**

- **7.NS.3 Apply the concepts of all four operations with rational numbers to solve real-world and mathematical problems.**
    - 7.NS.1 Extend prior knowledge of operations with positive rational numbers to add and to subtract all rational numbers and represent the sum or difference on a number line.
        - 7.NS.1a. Understand that the additive inverse of a number is its opposite and their sum is equal to zero.
        - 7.NS.1b. Understand that the sum of two rational numbers ($p+q$) represents a distance from p on the number line equal to |q| where the direction is indicated by the sign of q.
        - 7.NS.1c. Translate between the subtraction of rational numbers and addition using the additive inverse, $p-q=p+(-q)$.
        - 7.NS.1d. Demonstrate that the distance between two rational numbers on the number line is the absolute value of their difference.
        - 7.NS.1e. Apply mathematical properties (e.g., commutative, associative, distributive, or the properties of identity and inverse elements) to add and subtract rational numbers.
    - 7.NS.2 Extend prior knowledge of operations with positive rational numbers to multiply and to divide all rational numbers.
        - 7.NS.2a. Understand that the multiplicative inverse of a number is its reciprocal and their product is equal to one.
        - 7.NS.2b. Understand sign rules for multiplying rational numbers.
        - 7.NS.2c. Understand sign rules for dividing rational numbers and that a quotient of integers (with a non-zero divisor) is a rational number.
        - 7.NS.2d. Apply mathematical properties (e.g., commutative, associative, distributive, or the properties of identity and inverse elements) to multiply and divide rational numbers.
        - 7.NS.2e. Understand that some rational numbers can be written as integers and all rational numbers can be written as fractions or decimal numbers that terminate or repeat.

Ratios and Proportional Relationships

- **7.RP.3 Solve real-world and mathematical problems involving ratios and percentages using proportional reasoning (e.g., multi-step dimensional analysis, percent increase/decrease, tax).**
    - 7.RP.2 Identify and model proportional relationships given multiple representations, including tables, graphs, equations, diagrams, verbal descriptions, and real-world situations.
        - 7.RP.2a. Determine when two quantities are in a proportional relationship.
        - 7.RP.2b. Recognize or compute the constant of proportionality.
        - 7.RP.2c. Understand that the constant of proportionality is the unit rate.
        - 7.RP.2d. Use equations to model proportional relationships.
        - 7.RP.2e. Investigate the graph of a proportional relationship and explain the meaning of specific points (e.g., origin, unit rate) in the context of the situation.
    - 7.RP.1 Compute unit rates, including those involving complex fractions, with like or different units.
    - 7.GM.1 Determine the scale factor and translate between scale models and actual measurements (e.g., lengths, area) of real-world objects and geometric figures using proportional reasoning.

Expressions, Equations, and Equalities

- **7.EEI.3 Extend previous understanding of Order of Operations to solve multi- step real-world and mathematical problems involving rational numbers. Include fraction bars as a grouping symbol.**
    - 7.EEI.2 Recognize that algebraic expressions may have a variety of equivalent forms and determine an appropriate form for a given real-world situation.
    - 7.EEI.1 Apply mathematical properties (e.g., commutative, associative, distributive) to simplify and to factor linear algebraic expressions with rational coefficients.
    - 7.EEI.5 Understand and apply the laws of exponents (i.e., product rule, quotient rule, power to a power, product to a power, quotient to a power, zero power property) to simplify numerical expressions that include whole-number exponents.
    - 7.NS.1 Extend prior knowledge of operations with positive rational numbers to add and to subtract all rational numbers and represent the sum or difference on a number line.
        - 7.NS.1a. Understand that the additive inverse of a number is its opposite and their sum is equal to zero.

        - 7.NS.1b. Understand that the sum of two rational numbers ($p+q$) represents a distance from p on the number line equal to |q| where the direction is indicated by the sign of q.
        - 7.NS.1c. Translate between the subtraction of rational numbers and addition using the additive inverse, $p-q=p+(-q)$.
        - 7.NS.1d. Demonstrate that the distance between two rational numbers on the number line is the absolute value of their difference.
        - 7.NS.1e. Apply mathematical properties (e.g., commutative, associative, distributive, or the properties of identity and inverse elements) to add and subtract rational numbers.

- **7.NS.2 Extend prior knowledge of operations with positive rational numbers to multiply and to divide all rational numbers.**
    - 7.NS.2a. Understand that the multiplicative inverse of a number is its reciprocal and their product is equal to one.
    - 7.NS.2b. Understand sign rules for multiplying rational numbers.
    - 7.NS.2c. Understand sign rules for dividing rational numbers and that a quotient of integers (with a non-zero divisor) is a rational number.
    - 7.NS.2d. Apply mathematical properties (e.g., commutative, associative, distributive, or the properties of identity and inverse elements) to multiply and divide rational numbers.
    - 7.NS.2e. Understand that some rational numbers can be written as integers and all rational numbers can be written as fractions or decimal numbers that terminate or repeat.

- **7.EEI.4 Apply the concepts of linear equations and inequalities in one variable to real-world and mathematical situations.**
    - 7.EEI.4a. Write and fluently solve linear equations of the form $ax+b=c$ and $a(x+b)=c$ where a, b, and c are rational numbers.
    - 7.EEI.4b. Write and solve multi-step linear equations that include the use of the distributive property and combining like terms. Exclude equations that contain variables on both sides.
    - 7.EEI.4c. Write and solve two-step linear inequalities. Graph the solution set on a number line and interpret its meaning.
    - 7.EEI.4d. Identify and justify the steps for solving multi-step linear equations and two-step linear inequalities.
    - 7.NS.4 Understand and apply the concepts of comparing and ordering to rational numbers.

        - 7.NS.4a. Interpret statements using less than (<), greater than (>), less than or equal to (≤), greater than or equal to (≥), and equal to (=) as relative locations on the number line.
        - 7.NS.4b. Use concepts of equality and inequality to write and explain real-world and mathematical situations.

Geometry and Measurement

- **7.GM.6 Apply the concepts of two- and three-dimensional figures to real-world and mathematical situations.**
    - 7.GM.6a. Understand that the concept of area is applied to two-dimensional figures such as triangles, quadrilaterals, and polygons.
    - 7.GM.6b. Understand that the concepts of volume and surface area are applied to three-dimensional figures such as cubes, right rectangular prisms, and right triangular prisms.
    - 7.GM.6c. Decompose cubes, right rectangular prisms, and right triangular prisms into rectangles and triangles to derive the formulas for volume and surface area.
    - 7.GM.6d. Use the formulas for area, volume, and surface area appropriately.
    - 7.RP.3 Solve real-world and mathematical problems involving ratios and percentages using proportional reasoning (e.g., multi-step dimensional analysis, percent increase/decrease, tax).
    - 7.GM.3 Describe two-dimensional cross-sections of three-dimensional figures, specifically right rectangular prisms and right rectangular pyramids.

- **7.GM.5 Write equations to solve problems involving the relationships between angles formed by two intersecting lines, including supplementary, complementary, vertical, and adjacent.**
    - 7.GM.2 Construct triangles and special quadrilaterals using a variety of tools (e.g., freehand, ruler and protractor, technology).
        - 7.GM.2a. Construct triangles given all measurements of either angles or sides.
        - 7.GM.2b. Decide if the measurements determine a unique triangle, more than one triangle, or no triangle.
        - 7.GM.2c. Construct special quadrilaterals (i.e., kite, trapezoid, isosceles trapezoid, rhombus, parallelogram, rectangle) given specific parameters about angles or sides.

- **7.GM.4 Investigate the concept of circles.**
    - 7.GM.4a. Demonstrate an understanding of the proportional relationships between diameter, radius, and circumference of a circle.

- 7.GM.4b. Understand that the constant of proportionality between the circumference and diameter is equivalent to π.
- 7.GM.4c. Explore the relationship between circumference and area using a visual model.
- 7.GM.4d. Use the formulas for circumference and area of circles appropriately to solve real-world and mathematical problems.

Data Analysis and Probability

- **7.DSP.4 Compare the numerical measures of center (mean, median, mode) and variability (range, interquartile range, mean absolute deviation) from two random samples to draw inferences about the populations.**
  - 7.DSP.2 Draw inferences about a population by collecting multiple random samples of the same size to investigate variability in estimates of the characteristic of interest.
  - 7.DSP.1 Investigate concepts of random sampling.
    - 7.DSP.1a. Understand that a sample is a subset of a population and both possess the same characteristics.
    - 7.DSP.1b. Differentiate between random and non-random sampling.
    - 7.DSP.1c. Understand that generalizations from a sample are valid only if the sample is representative of the population.
    - 7.DSP.1d. Understand that random sampling is used to gather a representative sample and supports valid inferences about the population.

- **7.DSP.3 Visually compare the centers, spreads, and overlap of two displays of data (i.e., dot plots, histograms, box plots) that are graphed on the same scale and draw inferences about this data.**
  - 7.DSP.8 Extend the concepts of simple events to investigate compound events.
    - 7.DSP.8a. Understand that the probability of a compound event is between 0 and 1.
    - 7.DSP.8b. Identify the outcomes in a sample space using organized lists, tables, and tree diagrams.
    - 7.DSP.8c. Determine probabilities of compound events using organized lists, tables, and tree diagrams.
    - 7.DSP.8d. Design and use simulations to collect data and determine probabilities.
    - 7.DSP.8e. Compare theoretical and experimental probabilities for compound events.
  - 7.DSP.5 Investigate the concept of probability of chance events.
    - 7.DSP.5a. Determine probabilities of simple events.

        - 7.DSP.5b. Understand that probability measures likelihood of a chance event occurring.
        - 7.DSP.5c. Understand that the probability of a chance event is a number between 0 and 1.
        - 7.DSP.5d. Understand that a probability closer to 1 indicates a likely chance event.
        - 7.DSP.5e. Understand that a probability close to ½ indicates that a chance event is neither likely nor unlikely.
        - 7.DSP.5f. Understand that a probability closer to 0 indicates an unlikely chance event.
    - 7.DSP.6 Investigate the relationship between theoretical and experimental probabilities for simple events.
        - 7.DSP.6a. Determine approximate outcomes using theoretical probability.
        - 7.DSP.6b. Perform experiments that model theoretical probability.
        - 7.DSP.6c. Compare theoretical and experimental probabilities.
    - 7.DSP.7 Apply the concepts of theoretical and experimental probabilities for simple events.
        - 7.DSP.7a. Differentiate between uniform and non-uniform probability models (distributions).
        - 7.DSP.7b. Develop both uniform and non-uniform probability models.
        - 7.DSP.7c. Perform experiments to test the validity of probability models.

8th Grade
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The Number System

- **8.NS.1 Explore the real number system and its appropriate usage in real- world situations.**
    - 8.NS.1a. Recognize the differences between rational and irrational numbers.
    - 8.NS.1b. Understand that all real numbers have a decimal expansion.
    - 8.NS.1c. Model the hierarchy of the real number system, including natural, whole, integer, rational, and irrational numbers.
    - 8.EEI.2 Investigate concepts of square and cube roots.
        - 8.EEI.2a. Find the exact and approximate solutions to equations of the form $x^2=p$ and $x^3=p$ where p is a positive rational number.
        - 8.EEI.2b. Evaluate square roots of perfect squares.
        - 8.EEI.2c. Evaluate cube roots of perfect cubes.
        - 8.EEI.2d. Recognize that square roots of non-perfect squares are irrational.
    - 8.NS.2 Estimate and compare the value of irrational numbers by plotting them on a number line.

- **8.NS.3 Extend prior knowledge to translate among multiple representations of rational numbers (fractions, decimal numbers, percentages). Include the conversion of repeating decimal numbers to fractions.**

Expressions, Equations, and Equalities

- **8.EEI.4 Apply the concepts of decimal and scientific notation to solve real- world and mathematical problems.**
    - 8.EEI.4a. Multiply and divide numbers expressed in both decimal and scientific notation.
    - 8.EEI.4b. Select appropriate units of measure when representing answers in scientific notation.
    - 8.EEI.4c. Translate how different technological devices display numbers in scientific notation.
    - 8.EEI.3 Explore the relationship between quantities in decimal and scientific notation.
        - 8.EEI.3a. Express very large and very small quantities in scientific notation in the form $a\times10^b=p$ where $1\leq a<10$ and b is an integer.
        - 8.EEI.3b. Translate between decimal notation and scientific notation.
        - 8.EEI.3c. Estimate and compare the relative size of two quantities in scientific notation.

- **8.EEI.7 Extend concepts of linear equations and inequalities in one variable to more complex multi-step equations and inequalities in real-world and mathematical situations.**
    - 8.EEI.7a. Solve linear equations and inequalities with rational number coefficients that include the use of the distributive property, combining like terms, and variables on both sides.

- 8.EEI.7b. Recognize the three types of solutions to linear equations: one solution (x=a), infinitely many solutions (a=a), or no solutions (a=b).
- 8.EEI.7c. Generate linear equations with the three types of solutions.
- 8.EEI.7d. Justify why linear equations have a specific type of solution.
- 8.EEI.1 Understand and apply the laws of exponents (i.e., product rule, quotient rule, power to a power, product to a power, quotient to a power, zero power property, negative exponents) to simplify numerical expressions that include integer exponents.
- 8.EEI.2 Investigate concepts of square and cube roots.
    - 8.EEI.2a. Find the exact and approximate solutions to equations of the form x^2=p and x^3=p where p is a positive rational number.
- 8.EEI.5 Apply concepts of proportional relationships to real-world and mathematical situations.
    - 8.EEI.5a. Graph proportional relationships.
    - 8.EEI.5b. Interpret unit rate as the slope of the graph.
    - 8.EEI.5c. Compare two different proportional relationships given multiple representations, including tables, graphs, equations, diagrams, and verbal descriptions.
- 8.EEI.6 Apply concepts of slope and y-intercept to graphs, equations, and proportional relationships.
    - 8.EEI.6a. Explain why the slope, m, is the same between any two distinct points on a non-vertical line using similar triangles.
    - 8.EEI.6b. Derive the slope-intercept form (y=mx+b) for a non-vertical line.
    - 8.EEI.6c. Relate equations for proportional relationships (y=kx) with the slope-intercept form (y=mx+b) where b=0.
- 8.EEI.8 Investigate and solve real-world and mathematical problems involving systems of linear equations in two variables with integer coefficients and solutions.
    - 8.EEI.8a. Graph systems of linear equations and estimate their point of intersection.
    - 8.EEI.8b. Understand and verify that a solution to a system of linear equations is represented on a graph as the point of intersection of the two lines.
    - 8.EEI.8c. Solve systems of linear equations algebraically, including methods of substitution and elimination, or through inspection.
    - 8.EEI.8d. Understand that systems of linear equations can have one solution, no solution, or infinitely many solutions.

Functions

- **8.F.2 Compare multiple representations of two functions, including mappings, tables, graphs, equations, and verbal descriptions, in order to draw conclusions.**
    - 8.F.3 Investigate the differences between linear and nonlinear functions using multiple representations (i.e., tables, graphs, equations, and verbal descriptions).
        - 8.F.3a. Define an equation in slope-intercept form (y=mx+b) as being a linear function.

        - 8.F.3b. Recognize that the graph of a linear function has a constant rate of change.
        - 8.F.3c. Provide examples of nonlinear functions.

- **8.F.5 Apply the concepts of linear and nonlinear functions to graphs in real- world and mathematical situations.**
    - 8.F.5a. Analyze and describe attributes of graphs of functions (e.g., constant, increasing/decreasing, linear/nonlinear, maximum/minimum, discrete/continuous).
    - 8.F.5b. Sketch the graph of a function from a verbal description.
    - 8.F.5c. Write a verbal description from the graph of a function with and without scales.
    - 8.F.4 Apply the concepts of linear functions to real-world and mathematical situations.
        - 8.F.4a. Understand that the slope is the constant rate of change and the y-intercept is the point where $x = 0$.
        - 8.F.4b. Determine the slope and the y-intercept of a linear function given multiple representations, including two points, tables, graphs, equations, and verbal descriptions.
        - 8.F.4c. Construct a function in slope-intercept form that models a linear relationship between two quantities.
        - 8.F.4d. Interpret the meaning of the slope and the y-intercept of a linear function in the context of the situation.
        - 8.F.4e. Explore the relationship between linear functions and arithmetic sequences.
    - 8.F.1 Explore the concept of functions.
        - 8.F.1a. Understand that a function assigns to each input exactly one output.
        - 8.F.1b. Relate inputs (x-values or domain) and outputs (y-values or range) to independent and dependent variables.
        - 8.F.1c. Translate among the multiple representations of a function, including mappings, tables, graphs, equations, and verbal descriptions.
        - 8.F.1d. Determine if a relation is a function using multiple representations, including mappings, tables, graphs, equations, and verbal descriptions.
        - 8.F.1e. Graph a function from a table of values. Understand that the graph and table both represent a set of ordered pairs of that function.

Geometry and Measurement

- **8.GM.4 Apply the properties of transformations (rotations, reflections, translations, dilations).**
    - 8.GM.4a. Dilate geometric figures using scale factors that are positive rational numbers.
    - 8.GM.4b. Recognize that two-dimensional figures are only similar if a series of transformations can be performed to map the pre-image to the image.
    - 8.GM.4c. Given two similar figures, describe the series of transformations that justifies this similarity.

- 8.GM.4d. Use proportional reasoning to find the missing side lengths of two similar figures.
- 8.GM.1 Investigate the properties of rigid transformations (rotations, reflections, translations) using a variety of tools (e.g., grid paper, reflective devices, graphing paper, technology).
  - 8.GM.1a. Verify that lines are mapped to lines, including parallel lines.
  - 8.GM.1b. Verify that corresponding angles are congruent.
  - 8.GM.1c. Verify that corresponding line segments are congruent.
- 8.GM.2 Apply the properties of rigid transformations (rotations, reflections, translations).
  - 8.GM.2a. Rotate geometric figures 90, 180, and 270 degrees, both clockwise and counterclockwise, about the origin.
  - 8.GM.2b. Reflect geometric figures with respect to the x-axis and/or y-axis.
  - 8.GM.2c. Translate geometric figures vertically and/or horizontally.
  - 8.GM.2d. Recognize that two-dimensional figures are only congruent if a series of rigid transformations can be performed to map the pre-image to the image.
  - 8.GM.2e. Given two congruent figures, describe the series of rigid transformations that justifies this congruence.
- 8.GM.3 Investigate the properties of transformations (rotations, reflections, translations, dilations) using a variety of tools (e.g., grid paper, reflective devices, graphing paper, dynamic software).
  - 8.GM.3a. Use coordinate geometry to describe the effect of transformations on two-dimensional figures.
  - 8.GM.3b. Relate scale drawings to dilations of geometric figures.

- **8.GM.5 Extend and apply previous knowledge of angles to properties of triangles, similar figures, and parallel lines cut by a transversal.**
  - 8.GM.5a. Discover that the sum of the three angles in a triangle is 180 degrees.
  - 8.GM.5b. Discover and use the relationship between interior and exterior angles of a triangle.
  - 8.GM.5c. Identify congruent and supplementary pairs of angles when two parallel lines are cut by a transversal.
  - 8.GM.5d. Recognize that two similar figures have congruent corresponding angles.

- **8.GM.7 Apply the Pythagorean Theorem to model and solve real-world and mathematical problems in two and three dimensions involving right triangles.**
  - 8.GM.8 Find the distance between any two points in the coordinate plane using the Pythagorean Theorem.
  - 8.GM.6 Use models to demonstrate a proof of the Pythagorean Theorem and its converse.
  - 8.EEI.2 Investigate concepts of square and cube roots.

- **8.GM.9 Solve real-world and mathematical problems involving volumes of cones, cylinders, and spheres and the surface area of cylinders.**
    - 8.EEI.2 Investigate concepts of square and cube roots.

Data Analysis and Probability

- **8.DSP.3 Apply concepts of an approximate line of best fit in real-world situations.**
    - 8.DSP.3a. Find an approximate equation for the line of best fit using two appropriate data points.
    - 8.DSP.3b. Interpret the slope and intercept.
    - 8.DSP.3c. Solve problems using the equation.
    - 8.DSP.1 Investigate bivariate data.
        - 8.DSP.1a. Collect bivariate data.
        - 8.DSP.1b. Graph the bivariate data on a scatter plot.
        - 8.DSP.1c. Describe patterns observed on a scatter plot, including clustering, outliers, and association (positive, negative, no correlation, linear, nonlinear).
    - 8.DSP.2 Draw an approximate line of best fit on a scatter plot that appears to have a linear association and informally assess the fit of the line to the data points.
    - 8.DSP.4 Investigate bivariate categorical data in two-way tables.
        - 8.DSP.4a. Organize bivariate categorical data in a two-way table.
        - 8.DSP.4b. Interpret data in two-way tables using relative frequencies.
        - 8.DSP.4c. Explore patterns of possible association between the two categorical variables.
    - 8.DSP.5 Organize data in matrices with rational numbers and apply to real- world and mathematical situations.
        - 8.DSP.5a. Understand that a matrix is a way to organize data.
        - 8.DSP.5b. Recognize that a $m \times n$ matrix has m rows and n columns.
        - 8.DSP.5c. Add and subtract matrices of the same size.
        - 8.DSP.5d. Multiply a matrix by a scalar.

- **FA.ACE.1* Create and solve equations and inequalities in one variable that model real-world problems involving linear, quadratic, simple rational, and exponential relationships. Interpret the solutions and determine whether they are reasonable. (Limit to linear; quadratic; exponential with integer exponents.)**
    - FA.AREI.1* Understand and justify that the steps taken when solving simple equations in one variable create new equations that have the same solution as the original.
    - FA.AREI.3* Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
    - FA.FIF.8* Translate between different but equivalent forms of a function equation to reveal and explain different properties of the function. (Limit to linear; quadratic; exponential.)
      a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

- **FA.ACE.2* Create equations in two or more variables to represent relationships between quantities. Graph the equations on coordinate axes using appropriate labels, units, and scales. (Limit to linear; quadratic; exponential with integer exponents; direct and indirect variation.)**
    - FA.ACE.4* Solve literal equations and formulas for a specified variable including equations and formulas that arise in a variety of disciplines.
        - FA.AREI.1* Understand and justify that the steps taken when solving simple equations in one variable create new equations that have the same solution as the original.
        - FA.AREI.3* Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
    - FA.AREI.10* Explain that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane.
    - FA.FIF.8* Translate between different but equivalent forms of a function equation to reveal and explain different properties of the function. (Limit to linear; quadratic; exponential.)
        a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

- **FA.AREI.6* Solve systems of linear equations algebraically and graphically focusing on pairs of linear equations in two variables.**
    - FA.AREI.5 Justify that the solution to a system of linear equations is not changed when one of the equations is replaced by a linear combination of the other equation.
    - FA.AREI.6.a Solve systems of linear equations using the substitution method.
    - FA.AREI.6.b Solve systems of linear equations using linear combination.

- FA.AREI.11* Solve an equation of the form $f(x)=g(x)$ graphically by identifying the x-coordinate(s) of the point(s) of intersection of the graphs of $y= f(x)$ and $y=(x)$. (Limit to linear; quadratic; exponential.)

- **FA.AREI.12* Graph the solutions to a linear inequality in two variables.**

- **FA.ASE.1* Interpret the meanings of coefficients, factors, terms, and expressions based on their real-world contexts. Interpret complicated expressions as being composed of simpler expressions. (Limit to linear; quadratic; exponential.)**

- **FA.FBF.3* Describe the effect of the transformations (x), $f(x)+k$, $f(x+k)$, and combinations of such transformations on the graph of $y=f(x)$ for any real number k. Find the value of k given the graphs and write the equation of a transformed parent function given its graph. (Limit to linear; quadratic; exponential with integer exponents; vertical shift and vertical stretch.)**

- **FA.FIF.2* Evaluate functions and interpret the meaning of expressions involving function notation from a mathematical perspective and in terms of the context when the function describes a real-world situation.**
  - FA.FIF.1* Extend previous knowledge of a function to apply to general behavior and features of a function.
    c. Understand that the graph of a function labeled as f is the set of all ordered pairs (x, y) that satisfy the equation $y= f(x)$.
  - FA.FIF.8* Translate between different but equivalent forms of a function equation to reveal and explain different properties of the function. (Limit to linear; quadratic; exponential.)
    a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
  - FA.FLQE.5* Interpret the parameters in a linear or exponential function in terms of the context. (Limit to linear.)

- **FA.FIF.4* Interpret key features of a function that models the relationship between two quantities when given in graphical or tabular form. Sketch the graph of a function from a verbal description showing key features. Key features include intercepts; intervals where the function is increasing, decreasing, constant, positive, or negative; relative maximums and minimums; symmetries; end behavior and periodicity. (Limit to linear; quadratic; exponential.)**
  - FA.FIF.7* Graph functions from their symbolic representations. Indicate key features including intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior and periodicity. Graph simple cases by hand and use technology for complicated cases. (Limit to linear; quadratic; exponential only in the form $y=ax+k$.)

- FA.FIF.8* Translate between different but equivalent forms of a function equation to reveal and explain different properties of the function. (Limit to linear; quadratic; exponential.)
  a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

- **FA.FIF.5* Relate the domain and range of a function to its graph and, where applicable, to the quantitative relationship it describes. (Limit to linear; quadratic; exponential.)**
  - FA.FIF.1* Extend previous knowledge of a function to apply to general behavior and features of a function.
    a. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range.
      - FA.FIF.1* Extend previous knowledge of a function to apply to general behavior and features of a function.
        b. Represent a function using function notation and explain that $f\ (x)$ denotes the output of function f that corresponds to the input x.
  - FA.FLQE.5 * Interpret the parameters in a linear or exponential function in terms of the context. (Limit to linear.)

- **FA.FIF.9* Compare properties of two functions given in different representations such as algebraic, graphical, tabular, or verbal. (Limit to linear; quadratic; exponential.)**
  - FA.FLQE.3* Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or more generally as a polynomial function.

- **FA.FLQE.1* Distinguish between situations that can be modeled with linear functions or exponential functions by recognizing situations in which one quantity changes at a constant rate per unit interval as opposed to those in which a quantity changes by a constant percent rate per unit interval.**
  a. Prove that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals.

- **FA.NQ.1* Use units of measurement to guide the solution of multi-step tasks. Choose and interpret appropriate labels, units, and scales when constructing graphs and other data displays.**
  - FA.NQ.2* Label and define appropriate quantities in descriptive modeling contexts.
  - FA.NQ.3* Choose a level of accuracy appropriate to limitations on measurement when reporting quantities in context.

- **FA.NRNS.2* Use the definition of the meaning of rational exponents to translate between rational exponent and radical forms.**
    - FA.NRNS.1* Rewrite expressions involving simple radicals and rational exponents in different forms.
    - FA.NRNS.3 Explain why the sum or product of rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

- **FA.SPID.5* Analyze bivariate categorical data using two-way tables and identify possible associations between the two categories using marginal, joint, and conditional frequencies.**

- **FA.SPID.6* Using technology, create scatter plots and analyze those plots to compare the fit of linear, quadratic, or exponential models to a given data set. Select the appropriate model, fit a function to the data set, and use the function to solve problems in the context of the data.**
    - FA.SPID.7* Create a linear function to graphically model data from a real- world problem and interpret the meaning of the slope and intercept(s) in the context of the given problem.
    - FA.SPID.8* Using technology, compute and interpret the correlation coefficient of a linear fit.

- **FA.SPMJ.1* Understand statistics and sampling distributions as a process for making inferences about population parameters based on a random sample from that population.**
    - FA.SPMJ.2* Distinguish between experimental and theoretical probabilities. Collect data on a chance event and use the relative frequency to estimate the theoretical probability of that event. Determine whether a given probability model is consistent with experimental results.

- **FA.SPMD.5* Use probability to evaluate outcomes of decisions. Use probabilities to make fair decisions.**
    - FA.SPMD.4* Use probability to evaluate outcomes of decisions by finding expected values and determine if decisions are fair.
    - FA.SPMD.6* Analyze decisions and strategies using probability concepts.

- **IA.AAPR.1* Add, subtract, and multiply polynomials and understand that polynomials are closed under these operations.**

- **IA.ACE.1* Create and solve equations and inequalities in one variable that model real-world problems involving linear, quadratic, simple rational, and exponential relationships. Interpret the solutions and determine whether they are reasonable.**
    - IA.ACE.4* Solve literal equations and formulas for a specified variable including equations and formulas that arise in a variety of disciplines.
    - IA.AREI.4* Solve mathematical and real-world problems involving quadratic equations in one variable.
        a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x-h)^2=k$ that has the same solutions. Derive the quadratic formula from this form.
        b. Solve quadratic equations by inspection, taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a+bi$ for real numbers a and b.

- **IA.ACE.2* Create equations in two or more variables to represent relationships between quantities. Graph the equations on coordinate axes using appropriate labels, units, and scales.**
    - IA.FBF.3* Describe the effect of the transformations (x), $(x)+k$, $f(x+k)$, and combinations of such transformations on the graph of $y=f(x)$ for any real number k. Find the value of k given the graphs and write the equation of a transformed parent function given its graph.
    - IA.FIF.4* Interpret key features of a function that models the relationship between two quantities when given in graphical or tabular form. Sketch the graph of a function from a verbal description showing key features. Key features include intercepts; intervals where the function is increasing, decreasing, constant, positive, or negative; relative maximums and minimums; symmetries; end behavior and periodicity.
        - IA.ASE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. Determine the maximum or minimum value of a quadratic function by completing the square.
        - IA.FIF.7* Graph functions from their symbolic representations. Indicate key features including intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior and periodicity. Graph simple cases by hand and use technology for complicated cases.
    - IA.FIF.5* Relate the domain and range of a function to its graph and, where applicable, to the quantitative relationship it describes.

        - IA.FLQE.5* Interpret the parameters in a linear or exponential function in terms of the context.
    - IA.FIF.9* Compare properties of two functions given in different representations such as algebraic, graphical, tabular, or verbal.

- **IA.AREI.2* Solve simple rational and radical equations in one variable and understand how extraneous solutions may arise.**

- **IA.AREI.11* Solve an equation of the form $f(x)=g(x)$ graphically by identifying the x-coordinate(s) of the point(s) of intersection of the graphs of $y=(x)$ and $y=(x)$.**

- **IA.ASE.1* Interpret the meanings of coefficients, factors, terms, and expressions based on their real-world contexts. Interpret complicated expressions as being composed of simpler expressions.**
    - IA.ASE.2* Analyze the structure of binomials, trinomials, and other polynomials in order to rewrite equivalent expressions.

- **IA.ASE.3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.**
  a. Find the zeros of a quadratic function by rewriting it in equivalent factored form and explain the connection between the zeros of the function, its linear factors, the x-intercepts of its graph, and the solutions to the corresponding quadratic equation.

- **IA.FBF.1* Write a function that describes a relationship between two quantities.**
  b. Combine functions using the operations addition, subtraction, multiplication, and division to build new functions that describe the relationship between two quantities in mathematical and real-world situations.
    - IA.FBF.1 Write a function that describes a relationship between two quantities.
      a. Write a function that models a relationship between two quantities using both explicit expressions and a recursive process and by combining standard forms using addition, subtraction, multiplication and division to build new functions.
    - IA.FBF.2* Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.
    - IA.FIF.3* Define functions recursively and recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.
    - IA.ASE.2* Analyze the structure of binomials, trinomials, and other polynomials in order to rewrite equivalent expressions.

- **IA.FIF.6* Given a function in graphical, symbolic, or tabular form, determine the average rate of change of the function over a specified interval. Interpret the meaning of the average rate of change in a given context.**

- **IA.FLQE.2* Create symbolic representations of linear and exponential functions, including arithmetic and geometric sequences, given graphs, verbal descriptions, and tables**
    - IA.FBF.2* Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.
    - IA.FIF.3* Define functions recursively and recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.
    - IA.FIF.8 Translate between different but equivalent forms of a function equation to reveal and explain different properties of the function.
      b. Interpret expressions for exponential functions by using the properties of exponents.
    - IA.FLQE.5* Interpret the parameters in a linear or exponential function in terms of the context.

- **IA.NCNS.7* Solve quadratic equations in one variable that have complex solutions.**
    - IA.NCNS.1* Know there is a complex number b such that b^2=−1, and every complex number has the form $a + bi$ with a and b real.

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- **A1.AAPR.1* Add, subtract, and multiply polynomials and understand that polynomials are closed under these operations. (Limit to linear; quadratic.)**
    - A1.NRNS.3 Explain why the sum or product of rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

- **A1.ACE.1* Create and solve equations and inequalities in one variable that model real-world problems involving linear, quadratic, simple rational, and exponential relationships. Interpret the solutions and determine whether they are reasonable. (Limit to linear; quadratic; exponential with integer exponents.)**
    - A1.ACE.4* Solve literal equations and formulas for a specified variable including equations and formulas that arise in a variety of disciplines.
        - A1.AREI.3* Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
    - A1.AREI.1* Understand and justify that the steps taken when solving simple equations in one variable create new equations that have the same solution as the original.
    - A1.AREI.4. Solve mathematical and real-world problems involving quadratic equations in one variable.
        a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x-h)^2=k$ that has the same solutions. Derive the quadratic formula from this form.
        b. Solve quadratic equations by inspection, taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a+bi$ for real numbers a and b. (Limit to non-complex roots.)
    - A1.FIF.8* Translate between different but equivalent forms of a function equation to reveal and explain different properties of the function. (Limit to linear; quadratic; exponential.)
      a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

- **A1.ACE.2* Create equations in two or more variables to represent relationships between quantities. Graph the equations on coordinate axes using appropriate labels, units, and scales. (Limit to linear; quadratic; exponential with integer exponents; direct and indirect variation.)**
    - A1.FIF.8* Translate between different but equivalent forms of a function equation to reveal and explain different properties of the function. (Limit to linear; quadratic; exponential.)
    - A1.AREI.10* Explain that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane.

- **A1.AREI.6* Solve systems of linear equations algebraically and graphically focusing on pairs of linear equations in two variables.**
    - A1.AREI.5 Justify that the solution to a system of linear equations is not changed when one of the equations is replaced by a linear combination of the other equation.
    - A1.AREI.6.a Solve systems of linear equations using the substitution method.
    - A1.AREI.6.b Solve systems of linear equations using linear combination.
    - A1.AREI.11* Solve an equation of the form $(x)=g(x)$ graphically by identifying the x-coordinate(s) of the point(s) of intersection of the graphs of $y=f(x)$ and $y=(x)$ (Limit to linear; quadratic; exponential.)

- **A1.AREI.12* Graph the solutions to a linear inequality in two variables.**

- **A1.ASE.1* Interpret the meanings of coefficients, factors, terms, and expressions based on their real-world contexts. Interpret complicated expressions as being composed of simpler expressions. (Limit to linear; quadratic; exponential.)**
    - A1.ASE.2* Analyze the structure of binomials, trinomials, and other polynomials in order to rewrite equivalent expressions.

- **A1.ASE.3* Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.**
  a. Find the zeros of a quadratic function by rewriting it in equivalent factored form and explain the connection between the zeros of the function, its linear factors, the x-intercepts of its graph, and the solutions to the corresponding quadratic equation.
    - A1.FIF.8* Translate between different but equivalent forms of a function equation to reveal and explain different properties of the function. (Limit to linear; quadratic; exponential.)
        a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

- **A1.FBF.3* Describe the effect of the transformations (x), $(x)+k$, $f(x+k)$, and combinations of such transformations on the graph of $y=f(x)$ for any real number k. Find the value of k given the graphs and write the equation of a transformed parent function given its graph. (Limit to linear; quadratic; exponential with integer exponents; vertical shift and vertical stretch.)**

- **A1.FIF.2* Evaluate functions and interpret the meaning of expressions involving function notation from a mathematical perspective and in terms of the context when the function describes a real-world situation.**
    - A1.FIF.1 Extend previous knowledge of a function to apply to general behavior and features of a function.
        a. Understand that the graph of a function labeled as f is the set of all ordered pairs (x, y) that satisfy the equation $y=(x)$.

- A1.FIF.8* Translate between different but equivalent forms of a function equation to reveal and explain different properties of the function. (Limit to linear; quadratic; exponential.)
    a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
- A1.FLQE.5* Interpret the parameters in a linear or exponential function in terms of the context. (Limit to linear.)

- **A1.FIF.4* Interpret key features of a function that models the relationship between two quantities when given in graphical or tabular form. Sketch the graph of a function from a verbal description showing key features. Key features include intercepts; intervals where the function is increasing, decreasing, constant, positive, or negative; relative maximums and minimums; symmetries; end behavior and periodicity. (Limit to linear; quadratic; exponential.)**
    - A1.FIF.7* Graph functions from their symbolic representations. Indicate key features including intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior and periodicity. Graph simple cases by hand and use technology for complicated cases. (Limit to linear; quadratic; exponential only in the form $y=ax+k$.)
    - A1.FIF.8* Translate between different but equivalent forms of a function equation to reveal and explain different properties of the function. (Limit to linear; quadratic; exponential.)
      a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

- **A1.FIF.5* Relate the domain and range of a function to its graph and, where applicable, to the quantitative relationship it describes. (Limit to linear; quadratic; exponential.)**
    - A1.FIF.1 Extend previous knowledge of a function to apply to general behavior and features of a function.
        a. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range.
    - A1.FLQE.5* Interpret the parameters in a linear or exponential function in terms of the context. (Limit to linear.)

- **A1.FIF.6* Given a function in graphical, symbolic, or tabular form, determine the average rate of change of the function over a specified interval. Interpret the meaning of the average rate of change in a given context. (Limit to linear; quadratic; exponential.)**
    - A1.FLQE.1* Distinguish between situations that can be modeled with linear functions or exponential functions by recognizing situations in which one quantity

changes at a constant rate per unit interval as opposed to those in which a quantity changes by a constant percent rate per unit interval.
    a. Prove that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals.

- **A1.FIF.9* Compare properties of two functions given in different representations such as algebraic, graphical, tabular, or verbal. (Limit to linear; quadratic; exponential.)**
    - A1.FLQE.1* Distinguish between situations that can be modeled with linear functions or exponential functions by recognizing situations in which one quantity changes at a constant rate per unit interval as opposed to those in which a quantity changes by a constant percent rate per unit interval.
        a. Prove that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals.
    - A1.FLQE.3* Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or more generally as a polynomial function.

- **A1.FLQE.2* Create symbolic representations of linear and exponential functions, including arithmetic and geometric sequences, given graphs, verbal descriptions, and tables. (Limit to linear; exponential.)**
    - A1.FIF.1 Extend previous knowledge of a function to apply to general behavior and features of a function.
        b. Represent a function using function notation and explain that (x) denotes the output of function f that corresponds to the input x.

- **A1.NQ.1* Use units of measurement to guide the solution of multi-step tasks. Choose and interpret appropriate labels, units, and scales when constructing graphs and other data displays.**
    - A1.NQ.2* Label and define appropriate quantities in descriptive modeling contexts.
    - A1.NQ.3* Choose a level of accuracy appropriate to limitations on measurement when reporting quantities in context.

- **A1.NRNS.2* Use the definition of the meaning of rational exponents to translate between rational exponent and radical forms.**
    - A1.NRNS.1* Rewrite expressions involving simple radicals and rational exponents in different forms.

- **A1.SPID.6* Using technology, create scatter plots and analyze those plots to compare the fit of linear, quadratic, or exponential models to a given data set. Select the appropriate model, fit a function to the data set, and use the function to solve problems in the context of the data.**
    - A1.SPID.7* Create a linear function to graphically model data from a real-world problem and interpret the meaning of the slope and intercept(s) in the context of the given problem.

- A1.SPID.8* Using technology, compute and interpret the correlation coefficient of a linear fit.

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- **A2.AAPR.3 Graph polynomials identifying zeros when suitable factorizations are available and indicating end behavior. Write a polynomial function of least degree corresponding to a given graph. (Limit to polynomials with degrees 3 or less.)**
    - A2.AAPR.1* Add, subtract, and multiply polynomials and understand that polynomials are closed under these operations.
    - A2.FBF.3* Describe the effect of the transformations (x), $(x)+k$, $f(x+k)$, and combinations of such transformations on the graph of $y=f(x)$ for any real number k. Find the value of k given the graphs and write the equation of a transformed parent function given its graph.
    - A2.FIF.4* Interpret key features of a function that models the relationship between two quantities when given in graphical or tabular form. Sketch the graph of a function from a verbal description showing key features. Key features include intercepts; intervals where the function is increasing, decreasing, constant, positive, or negative; relative maximums and minimums; symmetries; end behavior and periodicity.
        - A2.FIF.7* Graph functions from their symbolic representations. Indicate key features including intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior and periodicity. Graph simple cases by hand and use technology for complicated cases.
        - A2.ASE.3* Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
          b. Determine the maximum or minimum value of a quadratic function by completing the square.
    - A2.FIF.5* Relate the domain and range of a function to its graph and, where applicable, to the quantitative relationship it describes.
    - A2.FIF.6* Given a function in graphical, symbolic, or tabular form, determine the average rate of change of the function over a specified interval. Interpret the meaning of the average rate of change in a given context.
    - A2.FIF.9* Compare properties of two functions given in different representations such as algebraic, graphical, tabular, or verbal.

- **A2.ACE.1* Create and solve equations and inequalities in one variable that model real-world problems involving linear, quadratic, simple rational, and exponential relationships. Interpret the solutions and determine whether they are reasonable.**
    - A2.ACE.4* Solve literal equations and formulas for a specified variable including equations and formulas that arise in a variety of disciplines.
    - A2.AREI.4* Solve mathematical and real-world problems involving quadratic equations in one variable.
        a. Solve quadratic equations by inspection, taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form

of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a + bi$ for real numbers a and b.

- A2.ASE.3* Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
  c. Use the properties of exponents to transform expressions for exponential functions.

- **A2.ACE.3 Use systems of equations and inequalities to represent constraints arising in real-world situations. Solve such systems using graphical and analytical methods, including linear programming. Interpret the solution within the context of the situation. (Limit to linear programming.)**
  - A2.ACE.2* Create equations in two or more variables to represent relationships between quantities. Graph the equations on coordinate axes using appropriate labels, units, and scales.

- **A2.AREI.2* Solve simple rational and radical equations in one variable and understand how extraneous solutions may arise.**
  - A2.ACE.4* Solve literal equations and formulas for a specified variable including equations and formulas that arise in a variety of disciplines.

- **A2.AREI.11* Solve an equation of the form $f(x)=g(x)$ graphically by identifying the x-coordinate(s) of the point(s) of intersection of the graphs of $y=f(x)$ and $y=(x)$**
  - A2.AREI.7 Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. Understand that such systems may have zero, one, two, or infinitely many solutions. (Limit to linear equations and quadratic functions.)

- **A2.ASE.1* Interpret the meanings of coefficients, factors, terms, and expressions based on their real-world contexts. Interpret complicated expressions as being composed of simpler expressions.**
  - A2.AAPR.1* Add, subtract, and multiply polynomials and understand that polynomials are closed under these operations.
  - A2.ASE.2* Analyze the structure of binomials, trinomials, and other polynomials in order to rewrite equivalent expressions.

- **A2.FBF.1* Write a function that describes a relationship between two quantities.**
  a. Write a function that models a relationship between two quantities using both explicit expressions and a recursive process and by combining standard forms using addition, subtraction, multiplication and division to build new functions.
  - A2.FBF.1 Write a function that describes a relationship between two quantities.
    b. Combine functions using the operations addition, subtraction, multiplication, and division to build new functions that describe the relationship between two quantities in mathematical and real-world situations.
  - A2.AAPR.1* Add, subtract, and multiply polynomials and understand that polynomials are closed under these operations.

- A2.ASE.2* Analyze the structure of binomials, trinomials, and other polynomials in order to rewrite equivalent expressions.

- **A2.FLQE.1* Distinguish between situations that can be modeled with linear functions or exponential functions by recognizing situations in which one quantity changes at a constant rate per unit interval as opposed to those in which a quantity changes by a constant percent rate per unit interval.**
  b. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
  - A2.FLQE.2* Create symbolic representations of linear and exponential functions, including arithmetic and geometric sequences, given graphs, verbal descriptions, and tables.
    - A2.ASE.3* Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
      c. Use the properties of exponents to transform expressions for exponential functions.
    - A2.FBF.2* Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.
    - A2.FIF.3* Define functions recursively and recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.
    - A2.FIF.8* Translate between different but equivalent forms of a function equation to reveal and explain different properties of the function.
      b. Interpret expressions for exponential functions by using the properties of exponents.
  - A2.FLQE.5* Interpret the parameters in a linear or exponential function in terms of the context.

- **A2.NCNS.7* Solve quadratic equations in one variable that have complex solutions.**
  - A2.NCNS.1* Know there is a complex number b such that b^2=−1, and every complex number has the form $a + bi$ with a and b real.

- **G.GCI.2* Identify and describe relationships among inscribed angles, radii, and chords; among inscribed angles, central angles, and circumscribed angles; and between radii and tangents to circles. Use those relationships to solve mathematical and real-world problems.**
    - G.GCI.3 Construct the inscribed and circumscribed circles of a triangle using a variety of tools, including a compass, a straightedge, and dynamic geometry software, and prove properties of angles for a quadrilateral inscribed in a circle.
    - G.GCI.4 Construct a tangent line to a circle through a point on the circle, and construct a tangent line from a point outside a given circle to the circle; justify the process used for each construction.
    - G.GCO.8* Prove, and apply in mathematical and real-world contexts, theorems about lines and angles, including the following:
        a. vertical angles are congruent;
        c. perpendicular lines form four right angles.
    - G.GCO.11* Construct geometric figures using a variety of tools, including a compass, a straightedge, dynamic geometry software, and paper folding, and use these constructions to make conjectures about geometric relationships.
    - G.GGPE.6 Given two points, find the point on the line segment between the two points that divides the segment into a given ratio.
    - G.GM.1* Use geometric shapes, their measures, and their properties to describe real-world objects.
    - G.GM.2 Use geometry concepts and methods to model real-world situations and solve problems using a model.

- **G.GM.2 Use geometry concepts and methods to model real-world situations and solve problems using a model.**
    - G.GCI.5* Derive the formulas for the length of an arc and the area of a sector in a circle and apply these formulas to solve mathematical and real-world problems.
    - G.GM.1* Use geometric shapes, their measures, and their properties to describe real-world objects.

- **G.GCO.3* Describe rotations and reflections that carry a regular polygon onto itself and identify types of symmetry of polygons, including line, point, rotational, and self-congruence, and use symmetry to analyze mathematical situations.**
    - G.GCO.2* Represent translations, reflections, rotations, and dilations of objects in the plane by using paper folding, sketches, coordinates, function notation, and dynamic geometry software, and use various representations to help understand the effects of simple transformations and their compositions.
    - G.GCO.4* Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.
    - G.GCO.6* Demonstrate that triangles and quadrilaterals are congruent by identifying a combination of translations, rotations, and reflections in various representations that move one figure onto the other.

- G.GSRT.5* Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

- **G.GCO.5* Predict and describe the results of transformations on a given figure using geometric terminology from the definitions of the transformations, and describe a sequence of transformations that maps a figure onto its image.**
  - G.GCO.2* Represent translations, reflections, rotations, and dilations of objects in the plane by using paper folding, sketches, coordinates, function notation, and dynamic geometry software, and use various representations to help understand the effects of simple transformations and their compositions.
  - G.GCO.4* Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.
  - G.GCO.6* Demonstrate that triangles and quadrilaterals are congruent by identifying a combination of translations, rotations, and reflections in various representations that move one figure onto the other.
  - G.GCO.7* Prove two triangles are congruent by applying the Side-Angle-Side, Angle-Side-Angle, Angle-Angle-Side, and Hypotenuse-Leg congruence conditions.
  - G.GSRT.5* Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

- **G.GCO.8* Prove, and apply in mathematical and real-world contexts, theorems about lines and angles, including the following:**
  c. any point on a perpendicular bisector of a line segment is equidistant from the endpoints of the segment;
  - G.GGPE.4* Use coordinates to prove simple geometric theorems algebraically.
  - G.GGPE.6 Given two points, find the point on the line segment between the two points that divides the segment into a given ratio.

- **G.GCO.9* Prove, and apply in mathematical and real-world contexts, theorems about the relationships within and among triangles, including the following:**
  b. base angles of isosceles triangles are congruent;
  c. the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length;
  d. the medians of a triangle meet at a point.
  - G.GCO.9* Prove, and apply in mathematical and real-world contexts, theorems about the relationships within and among triangles, including the following:
    a. measures of interior angles of a triangle sum to 180°;
  - G.GGPE.4* Use coordinates to prove simple geometric theorems algebraically.

- **G.GCO.10* Prove, and apply in mathematical and real-world contexts, theorems about parallelograms, including the following:**
  c. diagonals of a parallelogram bisect each other;
  d. rectangles are parallelograms with congruent diagonals;
  e. a parallelogram is a rhombus if and only if the diagonals are perpendicular.

- G.GCO.8* Prove, and apply in mathematical and real-world contexts, theorems about lines and angles, including the following:
  a. vertical angles are congruent;
  d. perpendicular lines form four right angles.
- G.GCO.10* Prove, and apply in mathematical and real-world contexts, theorems about parallelograms, including the following:
  a. opposite sides of a parallelogram are congruent;
  b. opposite angles of a parallelogram are congruent;
- G.GGPE.4* Use coordinates to prove simple geometric theorems algebraically.

- **G.GGMD.3* Apply surface area and volume formulas for prisms, cylinders, pyramids, cones, and spheres to solve problems and justify results. Include problems that involve algebraic expressions, composite figures, geometric probability, and real-world applications.**
  - G.GCO.1* Define angle, perpendicular line, parallel line, line segment, ray, circle, and skew in terms of the undefined notions of point, line, and plane. Use geometric figures to represent and describe real-world objects.
  - G.GCO.11* Construct geometric figures using a variety of tools, including a compass, a straightedge, dynamic geometry software, and paper folding, and use these constructions to make conjectures about geometric relationships.
  - G.GGMD.1* Explain the derivations of the formulas for the circumference of a circle, area of a circle, and volume of a cylinder, pyramid, and cone. Apply these formulas to solve mathematical and real-world problems.
  - G.GGMD.2 Explain the derivation of the formulas for the volume of a sphere and other solid figures using Cavalieri's principle.
  - G.GGMD.4 * Describe the shapes of two-dimensional cross-sections of three-dimensional objects and use those cross-sections to solve mathematical and real-world problems.
  - G.GM.1* Use geometric shapes, their measures, and their properties to describe real-world objects.
  - G.GM.2 Use geometry concepts and methods to model real-world situations and solve problems using a model.

- **G.GGPE.1* Understand that the standard equation of a circle is derived from the definition of a circle and the distance formula.**
  - G.GGPE.7* Use the distance and midpoint formulas to determine distance and midpoint in a coordinate plane, as well as areas of triangles and rectangles, when given coordinates.

- **G.GGPE.5* Analyze slopes of lines to determine whether lines are parallel, perpendicular, or neither. Write the equation of a line passing through a given point that is parallel or perpendicular to a given line. Solve geometric and real-world problems involving lines and slope.**
  - G.GCO.8* Prove, and apply in mathematical and real-world contexts, theorems about lines and angles, including the following:

b. when a transversal crosses parallel lines, alternate interior angles are congruent, alternate exterior angles are congruent, and consecutive interior angles are supplementary;
    - G.GM.1* Use geometric shapes, their measures, and their properties to describe real-world objects.
    - G.GM.2 Use geometry concepts and methods to model real-world situations and solve problems using a model.

- **G.GSRT.2* Use the definition of similarity to decide if figures are similar and justify decisions. Demonstrate that two figures are similar by identifying a combination of translations, rotations, reflections, and dilations in various representations that move one figure onto the other.**
    - G.GCI.1 Prove that all circles are similar.
    - G.GCO.2* Represent translations, reflections, rotations, and dilations of objects in the plane by using paper folding, sketches, coordinates, function notation, and dynamic geometry software, and use various representations to help understand the effects of simple transformations and their compositions.
    - G.GCO.4* Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.
    - G.GSRT.1 Understand a dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. Verify experimentally the properties of dilations given by a center and a scale factor. Understand the dilation of a line segment is longer or shorter in the ratio given by the scale factor.
        - G.GGPE.6 Given two points, find the point on the line segment between the two points that divides the segment into a given ratio.
    - G.GSRT.3* Prove that two triangles are similar using the Angle-Angle criterion and apply the proportionality of corresponding sides to solve problems and justify results.
    - G.GSRT.5* Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

- **G.GSRT.8* Solve right triangles in applied problems using trigonometric ratios and the Pythagorean Theorem.**
    - G.GM.1* Use geometric shapes, their measures, and their properties to describe real-world objects.
    - G.GM.2 Use geometry concepts and methods to model real-world situations and solve problems using a model.
    - G.GSRT.4* Prove, and apply in mathematical and real-world contexts, theorems involving similarity about triangles, including the following:
        a. A line drawn parallel to one side of a triangle divides the other two sides into parts of equal proportion.
        b. If a line divides two sides of a triangle proportionally, then it is parallel to the third side.
        c. The square of the hypotenuse of a right triangle is equal to the sum of squares of the other two sides.

- G.GSRT.6* Understand how the properties of similar right triangles allow the trigonometric ratios to be defined and determine the sine, cosine, and tangent of an acute angle in a right triangle.
- G.GSRT.7 Explain and use the relationship between the sine and cosine of complementary angles.

- **G.SPID.3* Summarize and represent data from a single data set. Interpret differences in shape, center, and spread in the context of the data set, accounting for possible effects of extreme data points (outliers).**
  - G.SPID.1* Select and create an appropriate display, including dot plots, histograms, and box plots, for data that includes only real numbers.
  - G.SPID.2* Use statistics appropriate to the shape of the data distribution to compare center and spread of two or more different data sets that include all real numbers.

Probability and Statistics
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- **PS.SPCR.1 Describe events as subsets of a sample space and**
  c. Represent sample spaces for compound events using Venn diagrams.
    - Use Venn diagrams to represent intersections, unions, and complements.
    - Relate intersections, unions, and complements to the words and, or, and not.

- **PS.SPCR.6 Calculate the conditional probability of an event A given event B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.**
    - PS.SPCR.3 Understand the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.
        - PS.SPCR.5 Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.
    - PS.SPCR.4 Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.

- **PS.SPCR.7 Apply the Addition Rule and the Multiplication Rule to determine probabilities, including conditional probabilities, and interpret the results in terms of the probability model.**
    - PS.SPCR.2 Use the multiplication rule to calculate probabilities for independent and dependent events. Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.
    - PS.SPCR.3 Understand the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.
        - PS.SPCR.5 Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.
    - PS.SPCR.4 Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.

- **PS.SPCR.8 Use permutations and combinations to solve mathematical and real-world problems, including determining probabilities of compound events. Justify the results.**
    - PS.SPCR.4 Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table

as a sample space to decide if events are independent and to approximate conditional probabilities.

- **PS.SPMJ.3 Plan and conduct a survey to answer a statistical question. Recognize how the plan addresses sampling technique, randomization, measurement of experimental error and methods to reduce bias.**
    - PS.SPMJ.1* Understand statistics and sampling distributions as a process for making inferences about population parameters based on a random sample from that population.
    - PS.SPMJ.2* Distinguish between experimental and theoretical probabilities. Collect data on a chance event and use the relative frequency to estimate the theoretical probability of that event. Determine whether a given probability model is consistent with experimental results.
    - PS.SPMJ.4 Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.
    - PS.SPMJ.5 Distinguish between experiments and observational studies. Determine which of two or more possible experimental designs will best answer a given research question and justify the choice based on statistical significance.

- **PS.SPMJ.6 Evaluate claims and conclusions in published reports or articles based on data by analyzing study design and the collection, analysis, and display of the data.**
    - PS.SPMJ.1* Understand statistics and sampling distributions as a process for making inferences about population parameters based on a random sample from that population.
    - PS.SPMJ.5 Distinguish between experiments and observational studies. Determine which of two or more possible experimental designs will best answer a given research question and justify the choice based on statistical significance.
    - PS.SPID.9 Differentiate between correlation and causation when describing the relationship between two variables. Identify potential lurking variables which may explain an association between two variables.

- **PS.SPID.7* Find linear models using median fit and regression methods to make predictions. Interpret the slope and intercept of a linear model in the context of the data.**
    - PS.SPID.1* Select and create an appropriate display, including dot plots, histograms, and box plots, for data that includes only real numbers.
    - PS.SPID.2* Use statistics appropriate to the shape of the data distribution to compare center and spread of two or more different data sets that include all real numbers.
    - PS.SPID.3* Summarize and represent data from a single data set. Interpret differences in shape, center, and spread in the context of the data set, accounting for possible effects of extreme data points (outliers).
    - PS.SPID.4 Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data

sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.
    - PS.SPID.5* Analyze bivariate categorical data using two-way tables and identify possible associations between the two categories using marginal, joint, and conditional frequencies.
    - PS.SPID.6* Using technology, create scatter plots and analyze those plots to compare the fit of linear, quadratic, or exponential models to a given data set. Select the appropriate model, fit a function to the data set, and use the function to solve problems in the context of the data.
    - PS.SPID.8* Compute using technology and interpret the correlation coefficient of a linear fit.

- **PS.SPID.10 Create residual plots and analyze those plots to compare the fit of linear, quadratic, and exponential models to a given data set. Select the appropriate model and use it for interpolation.**
    - PS.SPID.1* Select and create an appropriate display, including dot plots, histograms, and box plots, for data that includes only real numbers.
    - PS.SPID.2* Use statistics appropriate to the shape of the data distribution to compare center and spread of two or more different data sets that include all real numbers.
    - PS.SPID.3* Summarize and represent data from a single data set. Interpret differences in shape, center, and spread in the context of the data set, accounting for possible effects of extreme data points (outliers).
    - PS.SPID.5* Analyze bivariate categorical data using two-way tables and identify possible associations between the two categories using marginal, joint, and conditional frequencies.
    - PS.SPID.6* Using technology, create scatter plots and analyze those plots to compare the fit of linear, quadratic, or exponential models to a given data set. Select the appropriate model, fit a function to the data set, and use the function to solve problems in the context of the data.
    - PS.SPID.8* Compute using technology and interpret the correlation coefficient of a linear fit.

- **PS.SPMD.2 Calculate the expected value of a random variable as the mean of its probability distribution. Find expected values by assigning probabilities to payoff values. Use expected values to evaluate and compare strategies in real-world scenarios.**
    - PS.SPMD.1 Develop the probability distribution for a random variable defined for a sample space in which a theoretical probability can be calculated and graph the distribution.
    - PS.SPMD.3 Construct and compare theoretical and experimental probability distributions and use those distributions to find expected values.

- **PS.SPMD.5* Use probability to evaluate outcomes of decisions. Use probabilities to make fair decisions.**
    - PS.SPMD.1 Develop the probability distribution for a random variable defined for a sample space in which a theoretical probability can be calculated and graph the distribution.
    - PS.SPMD.3 Construct and compare theoretical and experimental probability distributions and use those distributions to find expected values.
    - PS.SPMD.4* Use probability to evaluate outcomes of decisions by finding expected values and determine if decisions are fair.
    - PS.SPMD.6* Analyze decisions and strategies using probability concepts.

- **PC.AAPR.3 Graph polynomials identifying zeros when suitable factorizations are available and indicating end behavior. Write a polynomial function of least degree corresponding to a given graph.**
    - PC.ASE.1 Interpret the meanings of coefficients, factors, terms, and expressions based on their real-world contexts. Interpret complicated expressions as being composed of simpler expressions.
    - PC.ASE.2 Analyze the structure of binomials, trinomials, and other polynomials in order to rewrite equivalent expressions.
    - PC.FBF.3 Describe the effect of the transformations (x), $(x)+k$, $f(x+k)$, and combinations of such transformations on the graph of $y=f(x)$ for any real number k. Find the value of k given the graphs and write the equation of a transformed parent function given its graph.
    - PC.FIF.4 Interpret key features of a function that models the relationship between two quantities when given in graphical or tabular form. Sketch the graph of a function from a verbal description showing key features. Key features include intercepts; intervals where the function is increasing, decreasing, constant, positive, or negative; relative maximums and minimums; symmetries; end behavior and periodicity.
    - PC.FIF.5 Relate the domain and range of a function to its graph and, where applicable, to the quantitative relationship it describes.
    - PC.FIF.6 Given a function in graphical, symbolic, or tabular form, determine the average rate of change of the function over a specified interval. Interpret the meaning of the average rate of change in a given context.
    - PC.AAPR.2 Know and apply the Division Theorem and the Remainder Theorem for polynomials.
    - PC.AAPR.5 Apply the Binomial Theorem to expand powers of binomials, including those with one and with two variables. Use the Binomial Theorem to factor squares, cubes, and fourth powers of binomials.
    - PC.NCNS.9 Know the Fundamental Theorem of Algebra and explain why complex roots of polynomials with real coefficients must occur in conjugate pairs.
        - PC.NCNS.7 Solve quadratic equations in one variable that have complex solutions.

- **PC.AAPR.6 Apply algebraic techniques to rewrite simple rational expressions in different forms; using inspection, long division, or, for the more complicated examples, a computer algebra system.**
    - PC.AAPR.7 Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

- **PC.AREI.7 Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. Understand that such systems may have zero, one, two, or infinitely many solutions.**
    - PC.AREI.11 Solve an equation of the form $(x)=g(x)$ graphically by identifying the x-coordinate(s) of the point(s) of intersection of the graphs of $y=f(x)$ and $y=g(x)$.

- **PC.AREI.9 Using technology for matrices of dimension 3 × 3 or greater, find the inverse of a matrix if it exists and use it to solve systems of linear equations.**
    - PC.AREI.8 Represent a system of linear equations as a single matrix equation in a vector variable.
    - PC.NVMQ.6* Use matrices to represent and manipulate data.
    - PC.NVMQ.7 Perform operations with matrices of appropriate dimensions including addition, subtraction, and scalar multiplication.
    - PC.NVMQ.8 Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.
    - PC.NVMQ.9 Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.

- **PC.ASE.4 Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems including applications to finance.**

- **PC.FBF.1 Write a function that describes a relationship between two quantities.**
  b. Combine functions using the operations addition, subtraction, multiplication, and division to build new functions that describe the relationship between two quantities in mathematical and real-world situations.

- **PC.FIF.7 Graph functions from their symbolic representations. Indicate key features including intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior and periodicity. Graph simple cases by hand and use technology for complicated cases.**
    - **a. Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.**
    - **b. Graph radical functions over their domain show end behavior.**
    - **c. Graph exponential and logarithmic functions, showing intercepts and end behavior.**
    - **d. Graph trigonometric functions, showing period, midline, and amplitude.**
        - PC.FBF.3 Describe the effect of the transformations (x), $(x)+k$, $f(x+k)$, and combinations of such transformations on the graph of $y=f(x)$ for any real number k. Find the value of k given the graphs and write the equation of a transformed parent function given its graph.

- PC.FIF.4 Interpret key features of a function that models the relationship between two quantities when given in graphical or tabular form. Sketch the graph of a function from a verbal description showing key features. Key features include intercepts; intervals where the function is increasing, decreasing, constant, positive, or negative; relative maximums and minimums; symmetries; end behavior and periodicity.
- PC.FIF.5 Relate the domain and range of a function to its graph and, where applicable, to the quantitative relationship it describes.
- PC.FIF.6 Given a function in graphical, symbolic, or tabular form, determine the average rate of change of the function over a specified interval. Interpret the meaning of the average rate of change in a given context.

- **PC. FLQE.4 Express a logarithm as the solution to the exponential equation, $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.**
  - PC.FBF.4 Understand that an inverse function can be obtained by expressing the dependent variable of one function as the independent variable of another, as f and g are inverse functions if and only if $(x) = y$ and $g(y) = x$, for all values of x in the domain of f and all values of y in the domain of g, and find inverse functions for one-to-one function or by restricting the domain.
    a. Use composition to verify one function is an inverse of another.
    b. If a function has an inverse, find values of the inverse function from a graph or table.
  - PC.FBF.5 Understand and verify through function composition that exponential and logarithmic functions are inverses of each other and use this relationship to solve problems involving logarithms and exponents.

- **PC.FT.4 Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.**
  - PC.FT.1 Understand that the radian measure of an angle is the length of the arc on the unit circle subtended by the angle.
  - PC.FT.2 Define sine and cosine as functions of the radian measure of an angle in terms of the x- and y-coordinates of the point on the unit circle corresponding to that angle and explain how these definitions are extensions of the right triangle definitions.
    a. Define the tangent, cotangent, secant, and cosecant functions as ratios involving sine and cosine.
    b. Write cotangent, secant, and cosecant functions as the reciprocals of tangent, cosine, and sine, respectively.
  - PC.FT.3 Use special triangles to determine geometrically the values of sine, cosine, tangent for $\frac{\pi}{3}$, $\frac{\pi}{4}$, and $\frac{\pi}{6}$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi - x$, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number.

- **PC.FT.5 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.**
    - PC.FBF.3 Describe the effect of the transformations (x), $(x)+k$, $f(x+k)$, and combinations of such transformations on the graph of $y=f(x)$ for any real number k. Find the value of k given the graphs and write the equation of a transformed parent function given its graph.
    - PC.FIF.4 Interpret key features of a function that models the relationship between two quantities when given in graphical or tabular form. Sketch the graph of a function from a verbal description showing key features. Key features include intercepts; intervals where the function is increasing, decreasing, constant, positive, or negative; relative maximums and minimums; symmetries; end behavior and periodicity.

- **PC.FT.7 Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.**
    - PC.FT.6 Define the six inverse trigonometric functions using domain restrictions for regions where the function is always increasing or always decreasing.

- **PC.FT.8 Justify the Pythagorean, even/odd, and cofunction identities for sine and cosine using their unit circle definitions and symmetries of the unit circle and use the Pythagorean identity to find sin$A$, cos$A$, or tan$A$, given sin$A$, cos$A$, or tan$A$, and the quadrant of the angle.**

- **PC.FT.9 Justify the sum and difference formulas for sine, cosine, and tangent and use them to solve problems.**

- **PC.GCI.5 Derive the formulas for the length of an arc and the area of a sector in a circle, and apply these formulas to solve mathematical and real-world problems.**

- **PC.GGPE.2 Use the geometric definition of a parabola to derive its equation given the focus and directrix.**

- **PC.GGPE.3 Use the geometric definition of an ellipse and of a hyperbola to derive the equation of each given the foci and points whose sum or difference of distance from the foci are constant.**

- **PC.GSRT.11 Use the Law of Sines and the Law of Cosines to solve for unknown measures of sides and angles of triangles that arise in mathematical and real-world problems.**
    - PC.GSRT.9 Derive the formula $A = \frac{1}{2}ab \sin C$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
    - PC.GSRT.10 Prove the Laws of Sines and Cosines and use them to solve problems.

- **PC.NCNS.5 Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation.**
    - PC.NCNS.3 Find the conjugate of a complex number in rectangular and polar forms and use conjugates to find moduli and quotients of complex numbers.
    - PC.NCNS.4 Graph complex numbers on the complex plane in rectangular and polar form and explain why the rectangular and polar forms of a given complex number represent the same number.
    - PC.NVMQ.11 Apply 2×2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.

- **PC.NCNS.6 Determine the modulus of a complex number by multiplying by its conjugate and determine the distance between two complex numbers by calculating the modulus of their difference.**

- **PC.NCNS.8 Extend polynomial identities to the complex numbers and use DeMoivre's Theorem to calculate a power of a complex number.**
    - PC.AAPR.4 Prove polynomial identities and use them to describe numerical relationships.
    - PC.AAPR.5 Apply the Binomial Theorem to expand powers of binomials, including those with one and with two variables. Use the Binomial Theorem to factor squares, cubes, and fourth powers of binomials.
    - PC.ASE.2 Analyze the structure of binomials, trinomials, and other polynomials in order to rewrite equivalent expressions.
    - PC.NCNS.2 Use the relation $i^2=-1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

- **PC.NVMQ.3 Represent and model with vector quantities. Solve problems involving velocity and other quantities that can be represented by vectors.**
    - PC.NVMQ.1 Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes.
    - PC.NVMQ.2 Represent and model with vector quantities. Use the coordinates of an initial point and of a terminal point to find the components of a vector.
    - PC.NVMQ.4 Perform operations on vectors.
      a. Add and subtract vectors using components of the vectors and graphically.
      b. Given the magnitude and direction of two vectors, determine the magnitude of their sum and of their difference.
    - PC.NVMQ.5 Multiply a vector by a scalar, representing the multiplication graphically and computing the magnitude of the scalar multiple.
    - PC.NVMQ.10 Multiply a vector by a matrix of appropriate dimension to produce another vector. Work with matrices as transformations of vectors.

Calculus
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- **C.LC.1 Understand the concept of a limit graphically, numerically, analytically, and contextually.**
  a. Estimate and verify limits using tables, graphs of functions, and technology.
    - C.LC.1 Understand the concept of a limit graphically, numerically, analytically, and contextually.
      b. Calculate limits, including one-sided limits, algebraically using direct substitution, simplification, rationalization, and the limit laws for constant multiples, sums, differences, products, and quotients.
      c. Calculate infinite limits and limits at infinity. Understand that infinite limits and limits at infinity provide information regarding the asymptotes of certain functions, including rational, exponential and logarithmic functions.

- **C.LC.2 Understand the definition and graphical interpretation of continuity of a function.**
  a. Apply the definition of continuity of a function at a point to solve problems.
    - C.LC.2 Understand the definition and graphical interpretation of continuity of a function.
      b. Classify discontinuities as removable, jump, or infinite. Justify that classification using the definition of continuity.
      c. Understand the Intermediate Value Theorem and apply the theorem to prove the existence of solutions of equations arising in mathematical and real-world problems.

- **C.D.1 Understand the concept of the derivative of a function geometrically, numerically, analytically, and verbally**
  a. Interpret the value of the derivative of a function as the slope of the corresponding tangent line.
  b. Interpret the value of the derivative as an instantaneous rate of change in a variety of real-world contexts such as velocity and population growth.
  c. Approximate the derivative graphically by finding the slope of the tangent line drawn to a curve at a given point and numerically by using the difference quotient.
    - C.D.1 Understand the concept of the derivative of a function geometrically, numerically, analytically, and verbally.
      d. Understand and explain graphically and analytically the relationship between differentiability and continuity.
      e. Explain graphically and analytically the relationship between the average rate of change and the instantaneous rate of change
      f. Understand the definition of the derivative and use this definition to determine the derivatives of various functions.

- **C.D.2 Apply the rules of differentiation to functions.**
  - **a. Know and apply the derivatives of constant, power, trigonometric, inverse trigonometric, exponential, and logarithmic functions.**
  - **b. Use the constant multiple, sum, difference, product, quotient, and chain rules to find the derivatives of functions.**
  - **c. Understand and apply the methods of implicit and logarithmic differentiation.**

- **C.D.3 Apply theorems and rules of differentiation to solve mathematical and real-world problems.**
  - **e. Solve a variety of real-world problems involving related rates, optimization, linear approximation, and rates of change.**
    - C.D.3 Apply theorems and rules of differentiation to solve mathematical and real-world problems.
      - a. Explain geometrically and verbally the mathematical and real-world meanings of the Extreme Value Theorem and the Mean Value Theorem.
      - b. Write an equation of a line tangent to the graph of a function at a point.
      - c. Explain the relationship between the increasing/decreasing behavior of f and the signs of f'. Use the relationship to generate a graph of f given the graph of f', and vice versa, and to identify relative and absolute extrema of f.
      - d. Explain the relationships among the concavity of the graph of f, the increasing/decreasing behavior of f' and the signs of f''. Use those relationships to generate graphs of f, f', and f'' given any one of them and identify the points of inflection of f.

- **C.I.1 Understand the concept of the integral of a function geometrically, numerically, analytically, and contextually.**
  - **b. Approximate definite integrals by calculating Riemann sums using left, right, and mid-point evaluations, and using trapezoidal sums.**
  - **c. Interpret the definite integral as a limit of Riemann sums.**
    - C.I.1 Understand the concept of the integral of a function geometrically, numerically, analytically, and contextually.
      - a. Explain how the definite integral is used to solve area problems.
      - d. Explain the relationship between the integral and derivative as expressed in both parts of the Fundamental Theorem of Calculus. Interpret the relationship in terms of rates of change.

- **C.I.2 Apply theorems and rules of integration to solve mathematical and real-world problems.**
  - **a. Apply the Fundamental Theorems of Calculus to solve mathematical and real-world problems.**
    - C.I.2 Apply theorems and rules of integration to solve mathematical and real-world problems.
      - b. Explain graphically and verbally the properties of the definite integral. Apply these properties to evaluate basic definite integrals.
      - c. Evaluate integrals using substitution.

Appendix A
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